# Quantum technology, group theory, phase space

Lecture 2, Peking University 2019

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SWINBURNE UNIVERSITY OF TECHNOLOGY

Mathematics based on Lie groups and Cartan spaces

- **Expand density matrix on a complete basis:**

$$\hat{\rho} = \int d\boldsymbol{\lambda} P\left(\boldsymbol{\lambda}\right) \hat{\Lambda}\left(\boldsymbol{\lambda}\right) ,$$

- Basis should have a unit trace:

$$\mathrm{Tr}\left[\hat{\Lambda}\left(\boldsymbol{\lambda}\right)\right] = 1 .$$

- **Normalized 'probability' may be real or complex:**

$$\int d\boldsymbol{\lambda} P\left(\boldsymbol{\lambda}\right) = 1 .$$

# +P PHASE-SPACE METHODS

## The positive P-representation expands in coherent state projectors

$$\hat{\rho} = \int P(\boldsymbol{\alpha},\boldsymbol{\beta})\hat{\Lambda}(\boldsymbol{\alpha},\boldsymbol{\beta}) d^{2M}\boldsymbol{\alpha} d^{2M}\boldsymbol{\beta}$$

$$\hat{\Lambda}(\boldsymbol{\alpha},\boldsymbol{\beta}) = \frac{|\boldsymbol{\alpha}\rangle\langle\boldsymbol{\beta}^*|}{\langle\boldsymbol{\beta}^*||\boldsymbol{\alpha}\rangle}$$

## Enlarged phase-space allows positive probabilities!

- Maps quantum states into 4M real coordinates: $\boldsymbol{\alpha},\boldsymbol{\beta} = \mathbf{p}+i\mathbf{x},\mathbf{p}'+i\mathbf{x}'$
- Double the size of a classical phase-space
- Exact mappings even for low occupations
- Advantage: Can represent entangled states

P. D. D. and C. W. Gardiner, J. Phys. A: Math. Gen. 13, 2353 (1980).

# 1: Boson sampling

Send N single photons through an M-channel photonic device

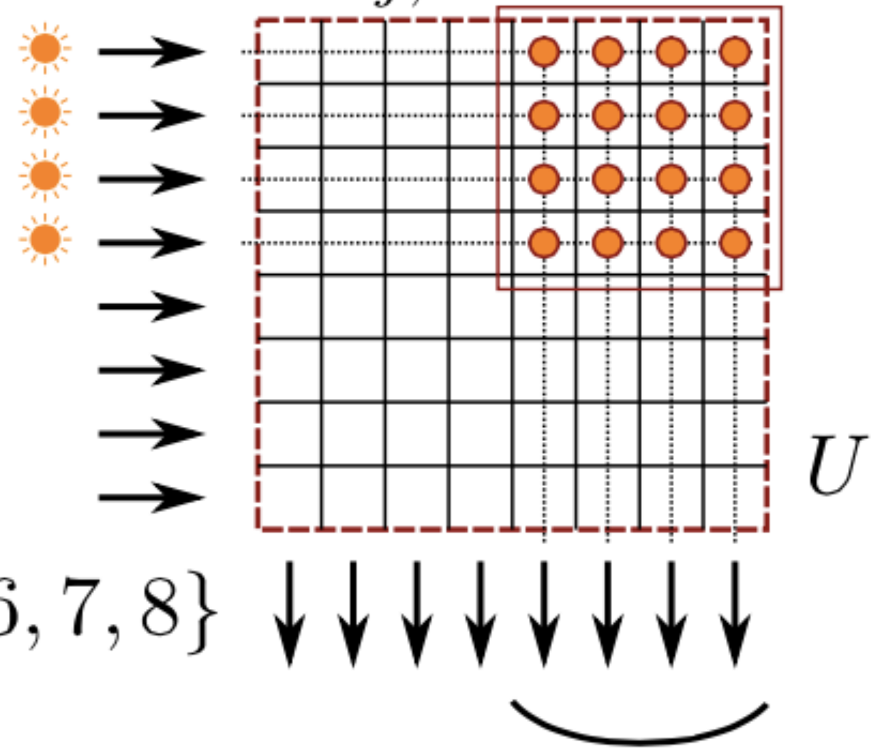
- Measure the output photon number distribution

**This solves the exponentially hard problem of generating random bits with permanent distribution**

- Matrix permanents are a '#P' hard problem, taking exponentially long times to compute at large N

# Boson sampling experiment: macroscopic quantum cat

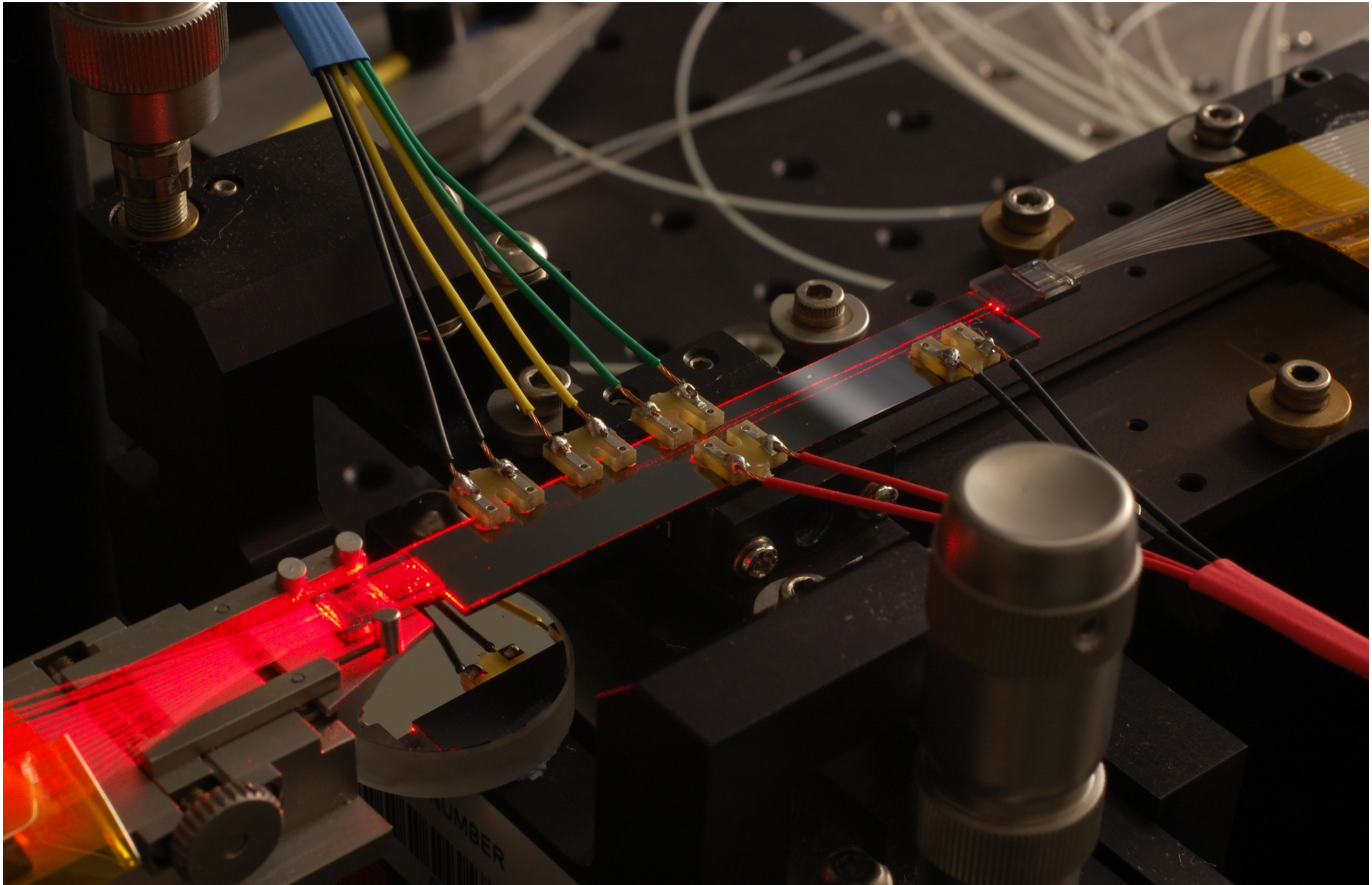
$\vec{j} = \{1, 2, 3, 4\}$ $\quad U_{\vec{j},\vec{k}} \equiv U_{N|M}$




$\vec{k} = \{5, 6, 7, 8\}$

$$\langle \hat{n}_{k_1} \ldots \hat{n}_{k_N} \rangle$$
$$= |\mathrm{perm} U_{\vec{j},\vec{k}}|^2$$

# Experiments:
# Oxford, Vienna, Queensland, Rome, USTC..



# Why is boson sampling hard?

## There are exponentially many interfering paths!

- The $N$-photon probability is a **matrix permanent**
- $$P = \left| \sum_{\sigma} \prod_{i} T_{i,\sigma(i)} \right|^2$$
- Here $T = \sqrt{1-\gamma}U$: $U$ is an $N \times N$ (sub)unitary, $\gamma$ a loss
- Standard methods take $N \times 2^N$ operations
- TRILLIONS of years for $N = 100$ at 1GFlop
- Impossible even on the largest supercomputers

**LARGEST PERMANENT EVER CALCULATED `EXACTLY':**

**N=50, TIANHE II, Wu et al,** Nat. Science Review, 5 715 (2018)

# Complex P-representation- 'complex weighted sampling'

## The $N$-mode, $N$-boson state,

$$P(\boldsymbol{\alpha},\boldsymbol{\beta}) = \frac{1}{(2\pi i)^{2N}} \prod_j \frac{e^{\alpha_j\beta_j} d\alpha_j d\beta_j}{(\alpha_j\beta_j)^2}$$

## Result for the output characteristic function:

$$\chi(\boldsymbol{\xi}) = \oint \dots \oint P(\boldsymbol{\alpha},\boldsymbol{\beta})\, e^{\boldsymbol{\xi}\cdot\boldsymbol{T}^*\boldsymbol{\beta} - \boldsymbol{\xi}^*\cdot\boldsymbol{T}\boldsymbol{\alpha}} d\boldsymbol{\alpha} d\boldsymbol{\beta}\,.$$

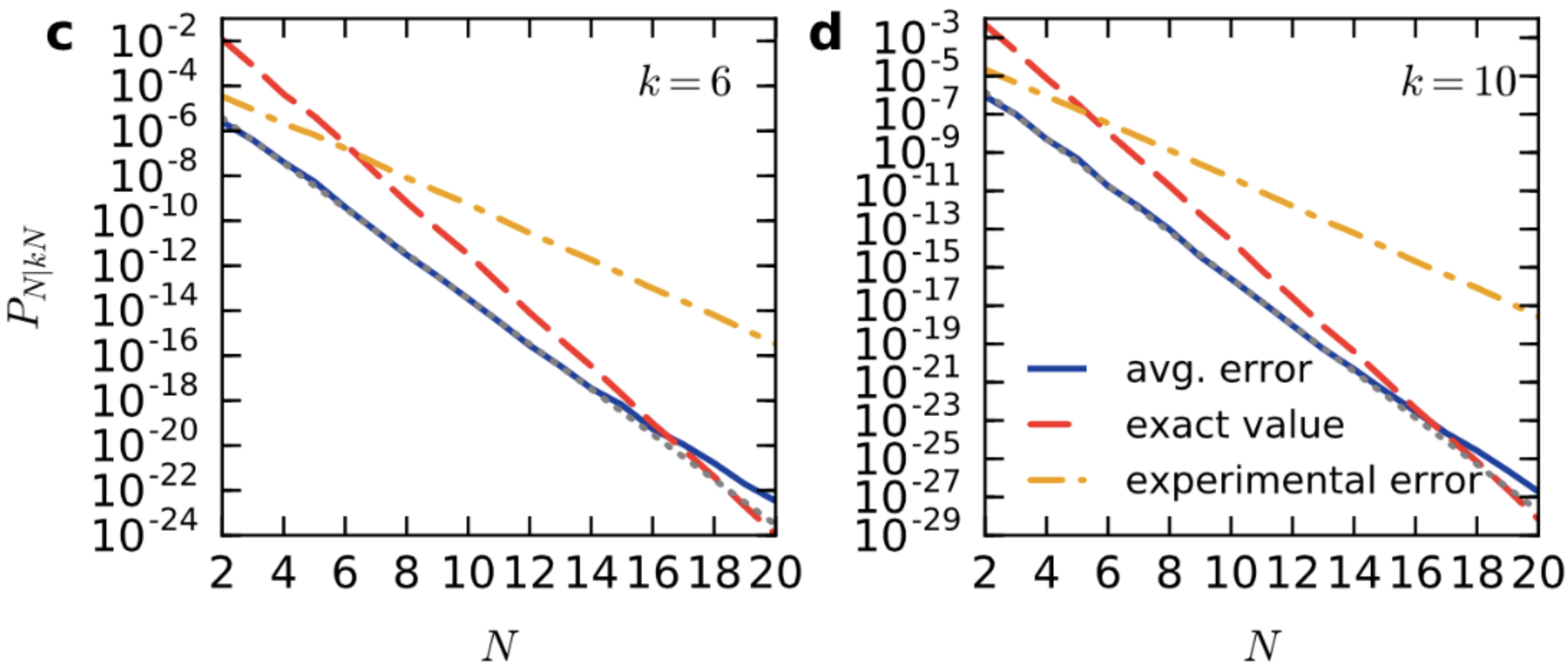
- Exact unitary averaged output depend on the *input* photon number $\hat{N}$:

$$\left\langle \chi^{(\mathrm{out})}(\boldsymbol{\xi}) \right\rangle_U = (M-1)! \sum_{j=0}^{M} \frac{\left(-t\,|\boldsymbol{\xi}|^2\right)^j \left\langle : \hat{N}^j : \right\rangle}{j!\,(M-1+j)!}$$

# Individual unitary simulation – possible at any size, but count rates get small

Randomly sample the complex-P contour integral;
simulates any permanent **much** better than experiment –

**Speed-up over a million times already at k=6, N=20**



# We can simulate any sub-unitary with better than experimental error!

## How do we interpret this result?

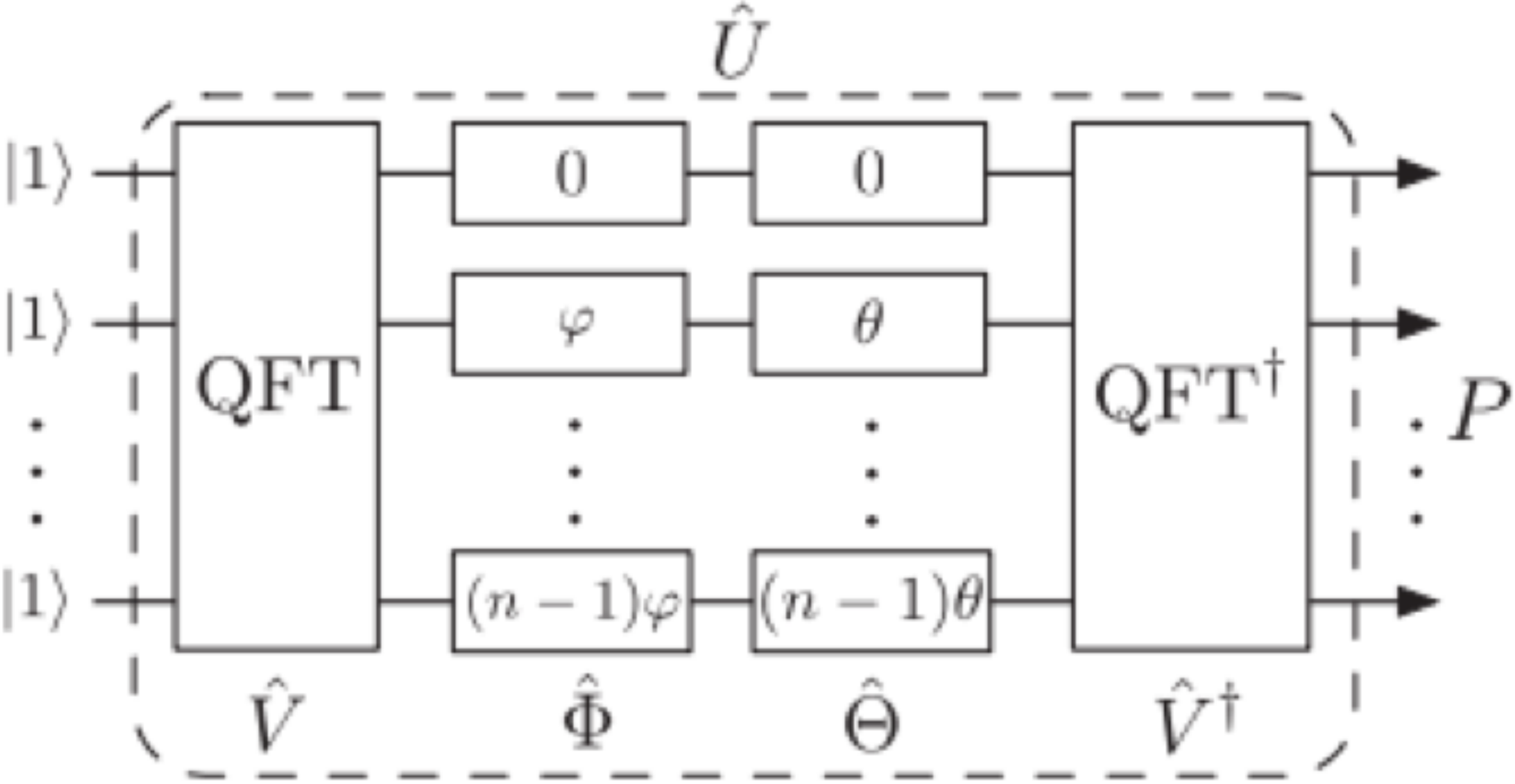
- Complex-P error in $|P|^2$ **decreases** rapidly with matrix-size $N$
- But, the experimental sampling error is proportional to $|P|$
- **We calculate $|P|^2$ better than experiment!**
- Don't generate a digital bitstream - doesn't solve a #P problem
- Can verify ANY possible N-th order correlation!
- Problem: correlations too small to measure at large $N$

# Is it useful? YES: Quantum Metrology!

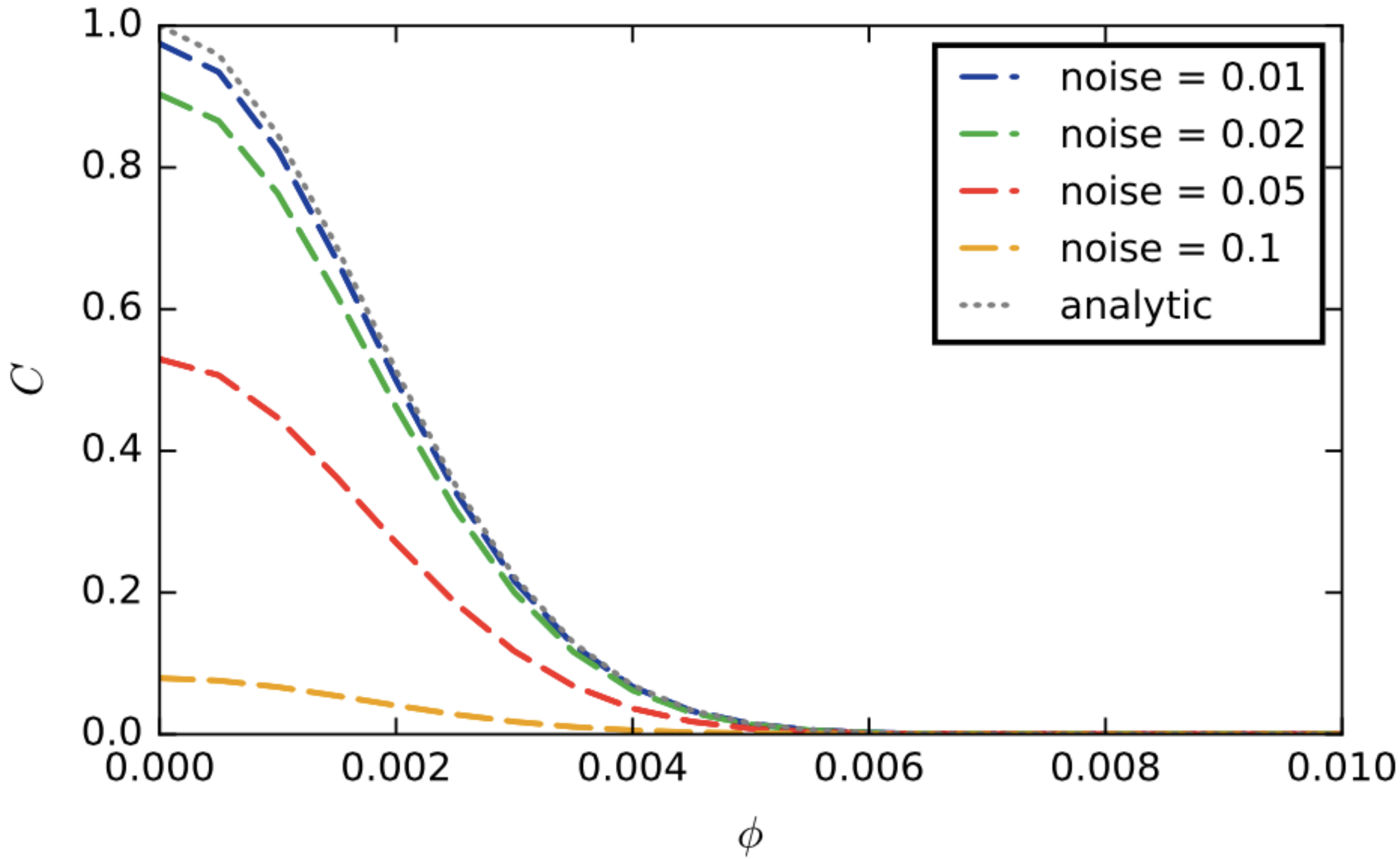
- Use a multichannel Quantum Fourier Transform
  - Enhances phase gradient measurement by N
  - Proposal by Rohde & Dowling groups
  - Ultrasensitive phase gradient measurements
- **How sensitive is this to phase decoherence?**
  - **Can compute 100x100 permanents**
  - Conventional supercomputer limits 50x50 (Tianhe II)
  - Would take trillions of years with standard methods

Opanchuk *et. al,* Optics Letters **44**, 343 (2019).

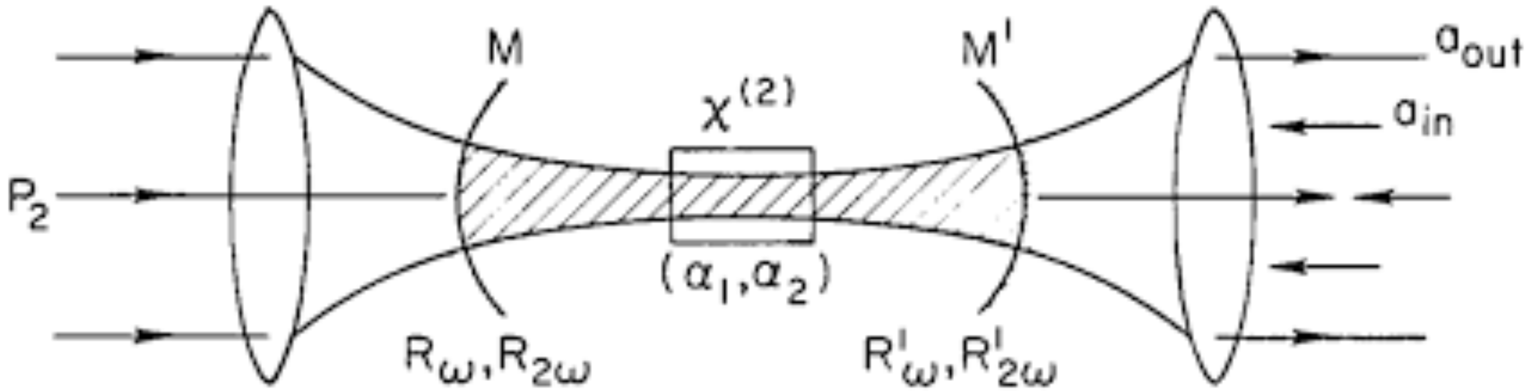
# Boson sampling enhanced metrology



# Strong fringes EVEN with added decoherence!



# 2: Quantum Circuit Cats




Exactly soluble model

Now used in LIGO, quantum cat experiments at Yale

# Hamiltonian

## Parametric interaction including $\chi^{(3)}$ nonlinearity

Pump ($k=2$) & downconverted field ($k=1$): $H = \hbar \sum_{n=1}^{6} \sum_{k=1}^{2} H_k^{(n)}$:

$$H_k^{(1)} = \left[\hat{\Gamma}_k a_k^{\dagger} + h.c.\right] + H_k^R \quad [Linear\ damping]$$

$$H_k^{(2)} = \left[\hat{\Gamma}_k^{(2)} a_k^{\dagger 2} + h.c.\right] + H_k^{R2} \quad [Nonlinear\ damping]$$

$$H_k^{(3)} + H_k^{(4)} = \omega_k a_k^{\dagger} a_k + \left[i\mathscr{E}_k a_k^{\dagger} e^{-ik\omega_p t} + h.c\right] \quad [Linear\ coupling]$$

$$H_k^{(5)} + H_1^{(6)} = \frac{\chi_k}{2} a_k^{\dagger 2} a_k^2 + \left[i\frac{\kappa}{2} a_2 a_1^{\dagger 2} + h.c.\right] \quad [Nonlinear\ coupling]$$

# Equivalent single-mode equation

## Two-mode problem mapped into an one-mode equivalent

Result of adiabatic elimination is a new complex FPE

$$\frac{\partial P_1}{\partial t} = \left\{ \frac{\partial}{\partial \alpha} \left[ \gamma\alpha - \mathscr{E}_1 - \varepsilon(\alpha)\,\alpha^+ \right] + \frac{1}{2}\frac{\partial^2}{\partial \alpha^2}\varepsilon(\alpha) + hc \right\} P_1 .$$

Here:

$$\varepsilon(\alpha) = \varepsilon - \chi\alpha^2$$

$$\varepsilon = \kappa\mathscr{E}_2/\gamma_2$$

$$\chi = \gamma_1^{(2)} + i\chi_1 + |\kappa|^2/2\gamma_2$$

# Exact complex P-function solution

## FPE has an exact steady-state solution

$$P_1(\vec{\alpha}) = N \exp[-\Phi(\vec{\alpha})]\,,$$

Introducing dimensionless parameters $c = (\gamma - \chi)/\chi$ and $\lambda_c = \varepsilon/\chi$,

$$\Phi(\vec{\alpha}) = -2\alpha^{+}\alpha - c\ln[\lambda_c - \alpha^2] - c^*\ln[\lambda_c^* - \alpha^{+2}],$$

**The steady-state probability distribution is given by**

$$P_S(\vec{\alpha}) = N(\lambda_c - \alpha^2)^c(\lambda_c^* - \alpha^{+2})^{c^*} e^{2\alpha^{+}\alpha}.$$

**Feng-Xiao Sun, et. al, New Journal of Physics, 21, 093035 (2019); Physical Review A 100, 033827 (2019).**

# Dimensionless variables

## Scale parameters to get universal behaviour

- Let: $\beta = \alpha/\sqrt{\lambda_c}$ and $\beta^+ = \alpha^+/\sqrt{\lambda_c^*}$.
- We introduce $\lambda = |\lambda_c|$ and $\lambda(\beta) = \lambda\left(1 - \beta^2\right)$.

In the scaled coherent space:

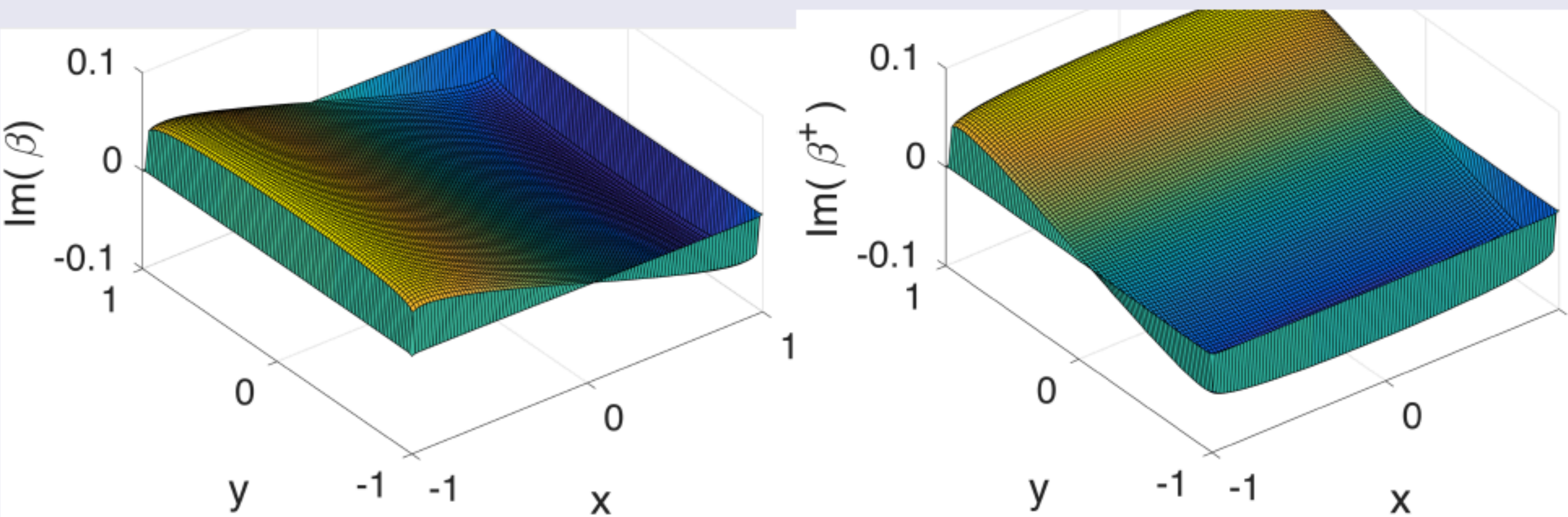
$$P_S\left(\vec{\beta}\right) = N(1-\beta^2)^c(1-\beta^{+2})^{c^*}e^{2\lambda\beta^+\beta}.$$

Boundaries: probability vanishes at $\beta = \pm 1$, $\beta^+ = \pm 1$.

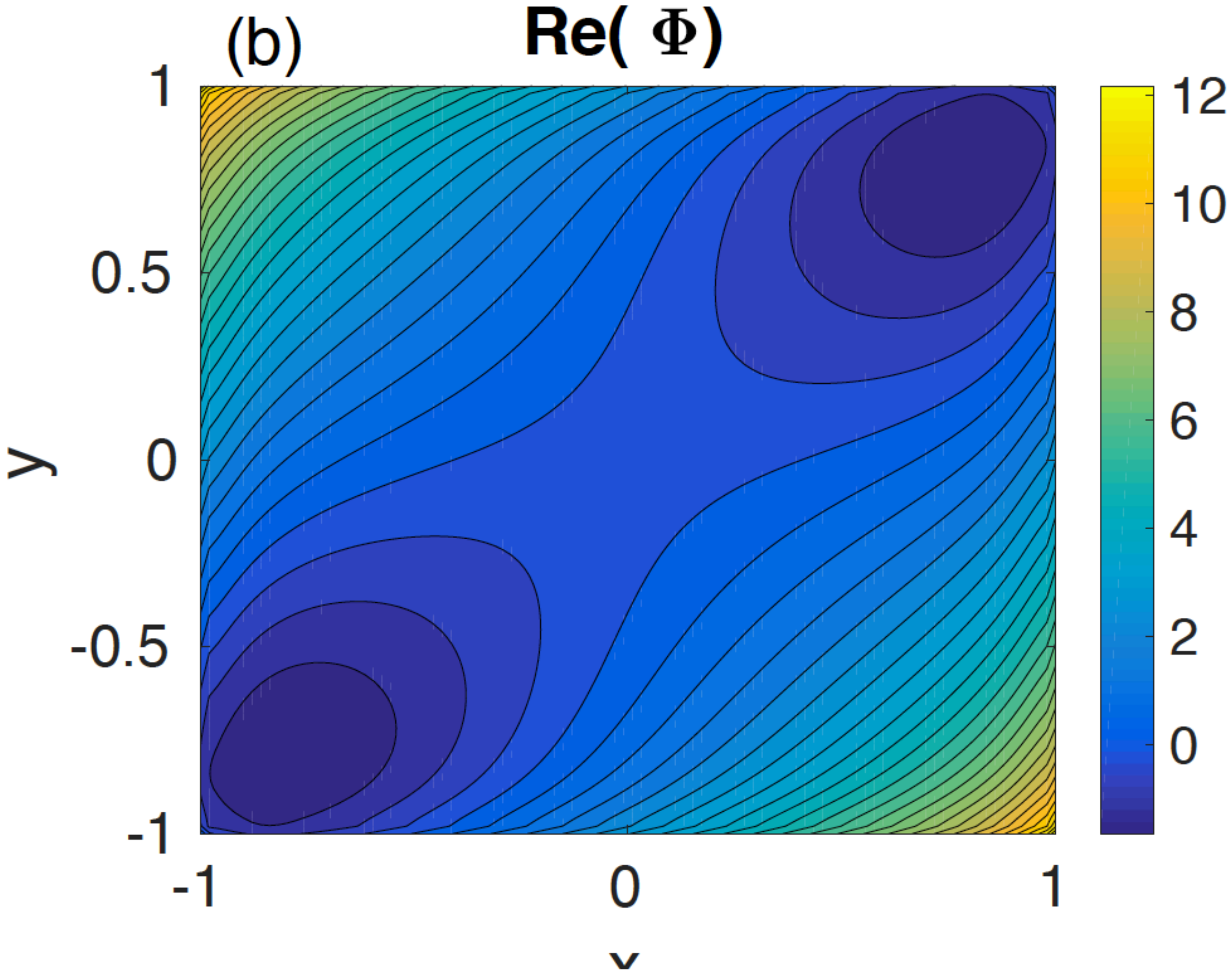
# Manifold of coherent amplitudes

$$\beta = x + ix\tan(\varphi)\cos^p(x\pi/2)\cos^p(y\pi/2),$$
$$\beta^+ = y - iy\tan(\varphi)\cos^p(x\pi/2)\cos^p(y\pi/2).$$

## Manifold is a 2D surface in 4D phase-space



# Potential for tunneling



# Tunneling: How to escape a local minimum

## Swanson-Landauer theory, with complex potentials

Analytic formula valid in the large barrier limit

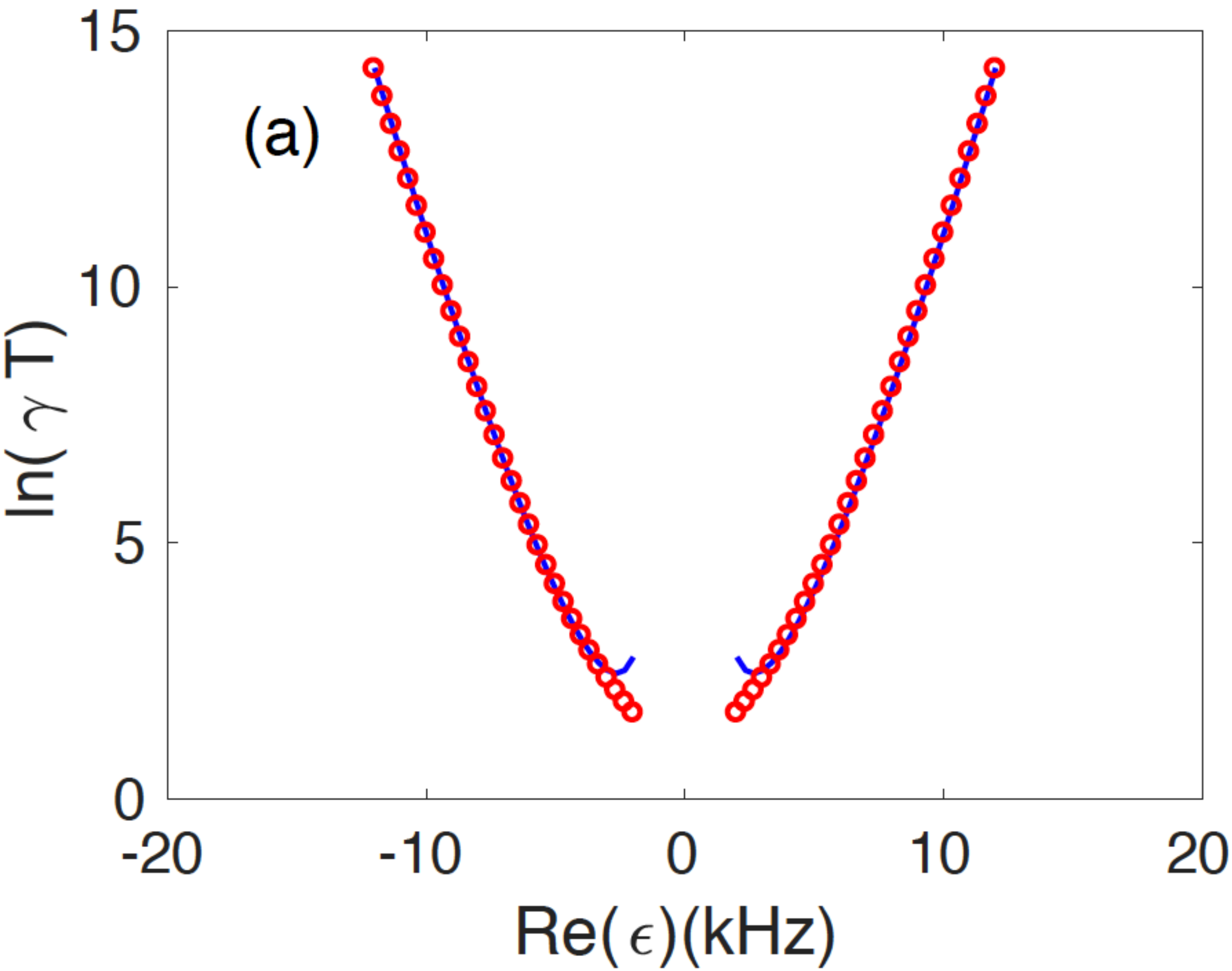
$$T = \frac{2\pi}{|\chi|\cos 2\phi}\left[\frac{-\Phi_{vv}^{(o)}}{\Phi_{uu}^{(o)}\Phi_{uu}^{(c)}\Phi_{vv}^{(c)}}\right]^{\frac{1}{2}}\exp(\Phi^{(o)}-\Phi^{(c)})$$

Simplest case: no anharmonic term ($Im(\chi)=0$), let $\bar{c}=c+1/2$:

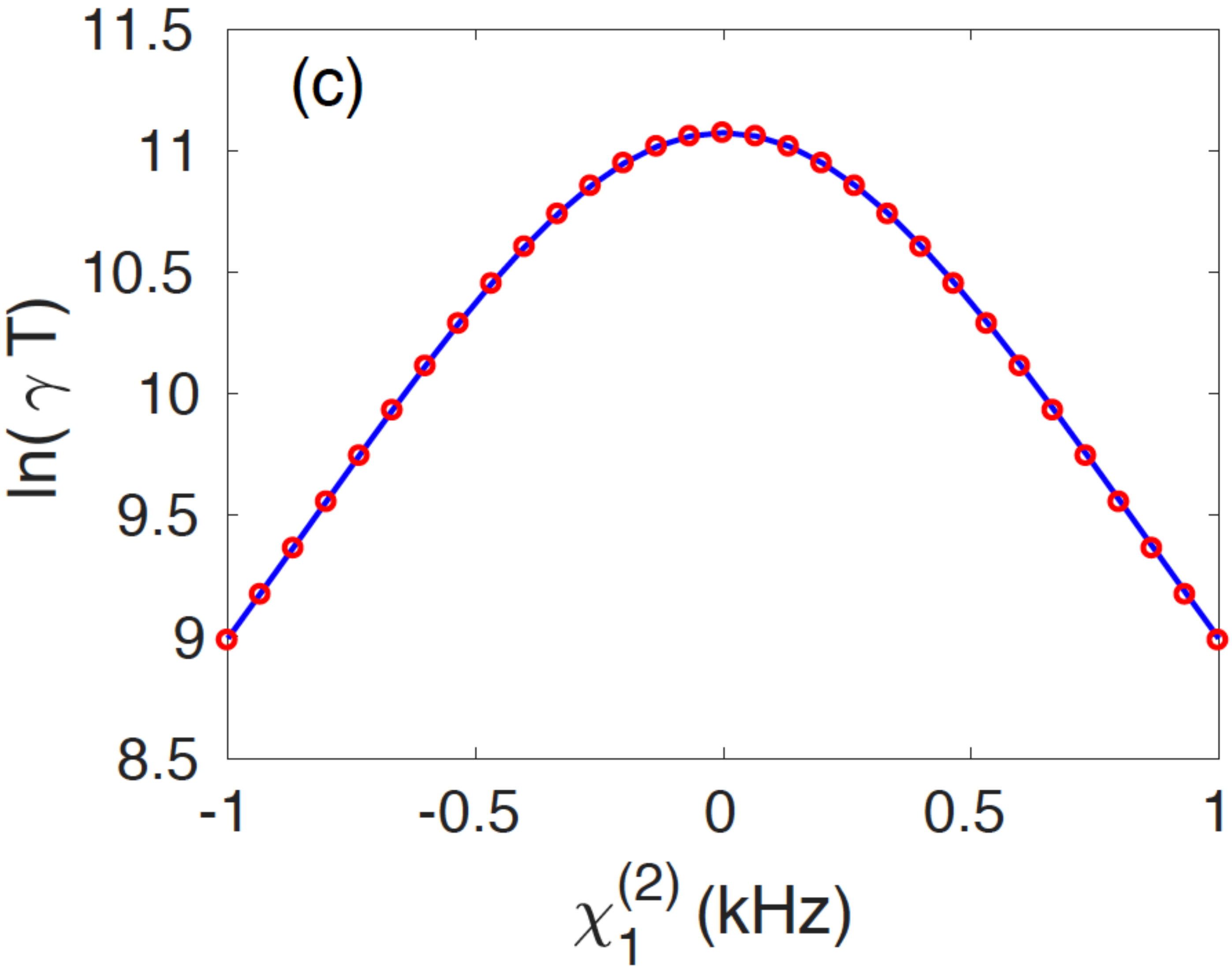
$$T = \frac{\pi}{|\chi|}\left[\frac{\lambda+\bar{c}}{\lambda(\lambda-\bar{c})^2}\right]^{\frac{1}{2}}\exp\left\{2\left[\lambda-\bar{c}-\bar{c}\ln\left(\frac{\lambda}{\bar{c}}\right)\right]\right\},$$

Can also calculate numerically with number states - red circles below

# Tunneling rates versus pump amplitude



# Tunneling rates versus anharmonicity



# Steady-state moments

- Exact solution

$$I^{ex}_{nn'} \propto \sum_m \frac{(2\lambda)^m}{m!}(-\sqrt{\lambda_c})^{n'} {}_2F_1(-m-n', c+1, 2c+2, 2)$$
$$\times(-\sqrt{\lambda_c^*})^n {}_2F_1(-m-n, c^*+1, 2c^*+1, 2)$$

- Wolinsky & Carmichael (PRL) :

$$I^{\delta}_{nn'} \propto e^{2\lambda}\left[(\sqrt{\lambda_c})^{n'}(\sqrt{\lambda_c^*})^n + (-\sqrt{\lambda_c})^{n'}(-\sqrt{\lambda_c^*})^n\right]$$
$$+e^{-2\lambda}\left[(-\sqrt{\lambda_c})^{n'}(\sqrt{\lambda_c^*})^n + (\sqrt{\lambda_c})^{n'}(-\sqrt{\lambda_c^*})^n\right].??$$

- Schrödinger Cat:

$$I^{\delta}_{nn'} = e^{\lambda}\left[(\sqrt{\lambda_c})^{n'}(\sqrt{\lambda_c^*})^n + (-\sqrt{\lambda_c})^{n'}(-\sqrt{\lambda_c^*})^n\right]$$
$$+e^{-\lambda}\left[(-\sqrt{\lambda_c})^{n'}(\sqrt{\lambda_c^*})^n + (\sqrt{\lambda_c})^{n'}(-\sqrt{\lambda_c^*})^n\right].$$

# Schrödinger cats only form as transients!

## Cats CAN form, but not steady-state

- Steady-state solution exists at strong coupling
- For $\Re(c) < 0$ get a pole at the boundary
- Weak coupling manifold is unstable
- Must change to a new manifold

- Steady-state Wigner is positive (Reid&Yurke) $\Longrightarrow$ no cat

## Work on transient cats-

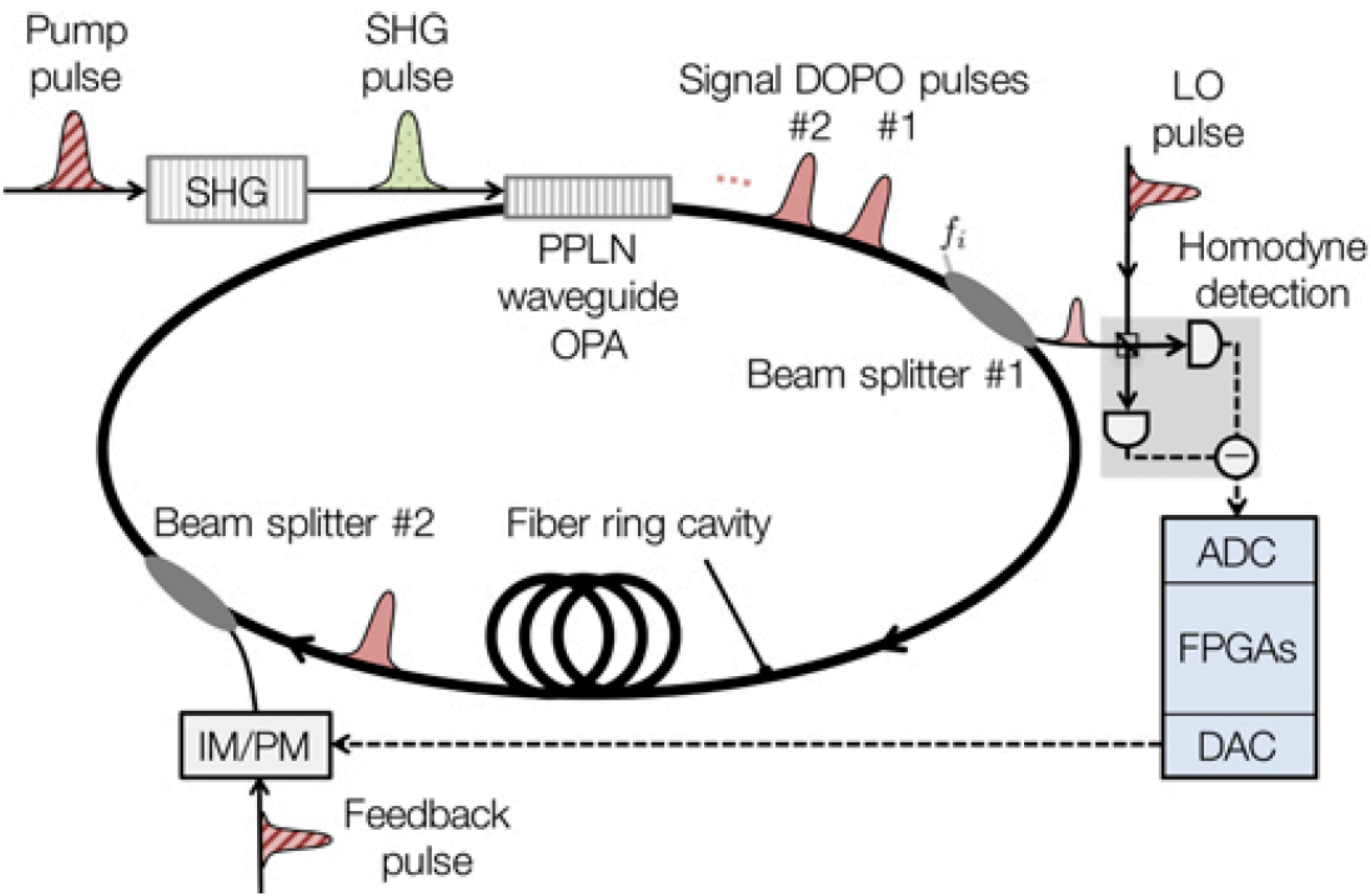

- M. Reid, B. Yurke, Phys. Rev. A 46, 4131 (1992).
- L. Krippner, W. Munro, M. Reid, Phys. Rev. A 50, 4330 (1994).
- W. Munro, M. Reid, Phys. Rev. A 52, 2388 (1995).

# Applications to quantum computers

Universal quantum computers have decoherence, scaling problems

Alternative: Dedicated hardware for NP-hard problems

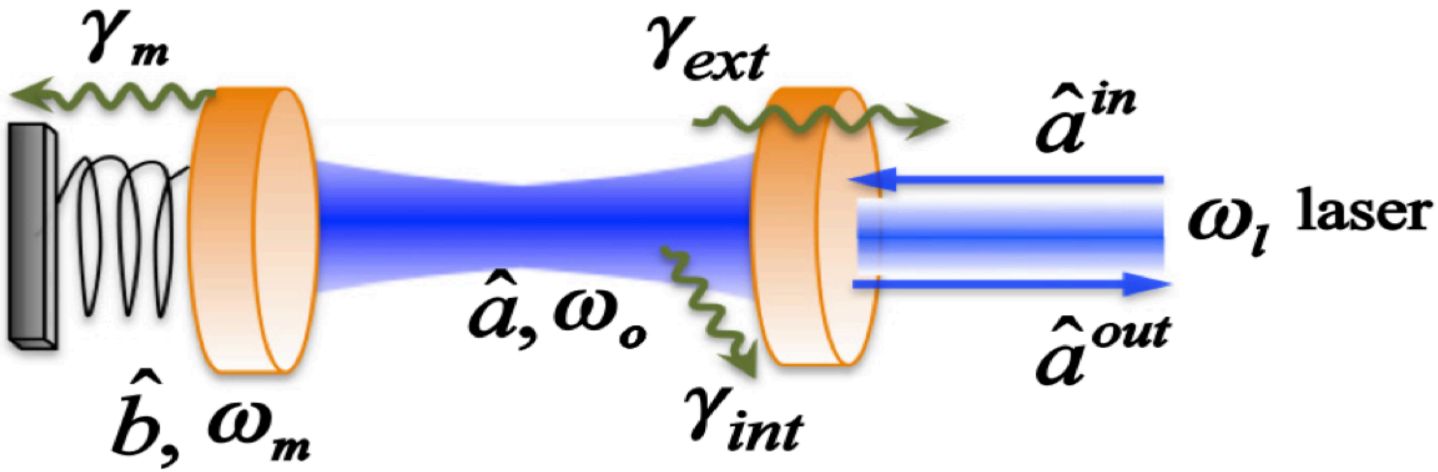
# The Ising machine: a paramp network



# CIM Simulations

- Can be simulated with complex/positive P
- Already reaches 2000 qubits in size
- Solves 100 times larger problems than D-wave
- NTT Phi-lab opened in San Jose in July
- Joint research program with SUT
- New techniques for deep quantum regime

# 3: Optomechanical Cats




- First principles quantum simulations
- Nonlinear model
- Entanglement agrees with experiment

# Hamiltonian

$$\hat{H}/\hbar = \delta \hat{a}^{\dagger}\hat{a} + \omega_m \hat{b}^{\dagger}\hat{b} + \chi \hat{a}^{\dagger}\hat{a}(\hat{b} + \hat{b}^{\dagger}) + iE(t)(\hat{a}^{\dagger} - \hat{a}) + \hat{H}_r .$$

Standard model for nonlinear optomechanical Hamiltonian

A. F. Pace, M. J. Collett, and D. F. Walls, Phys. Rev. A 47, 3173 (1993).

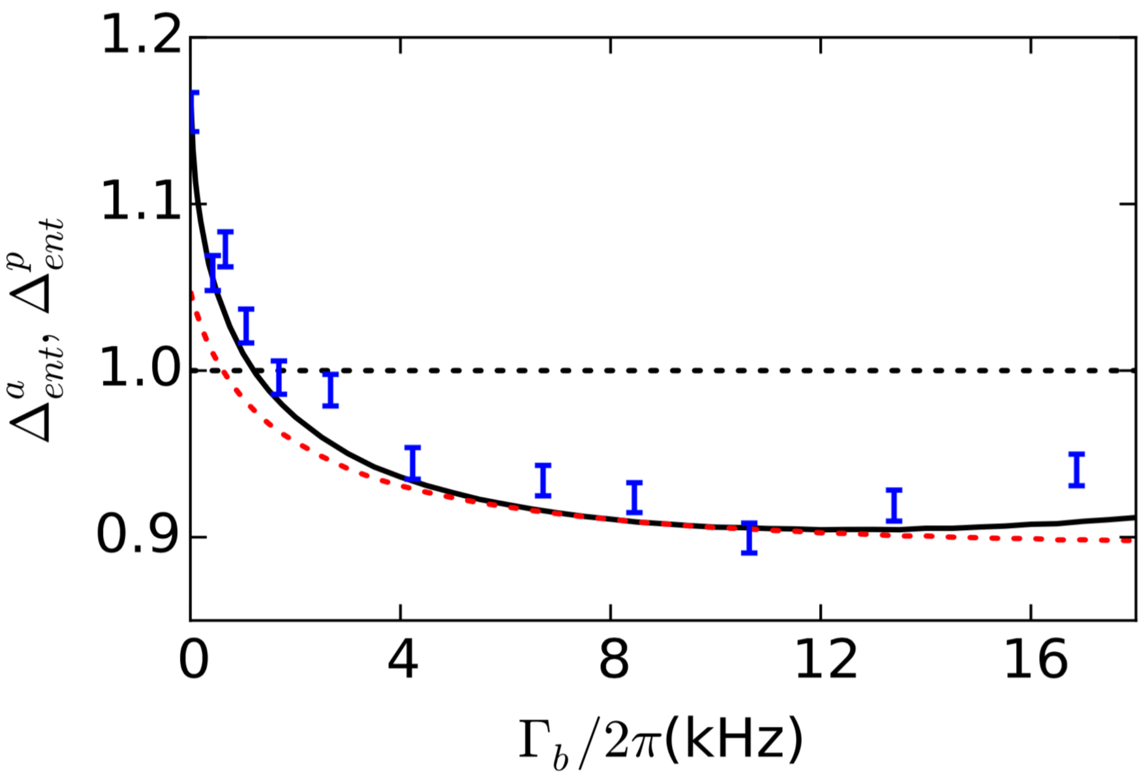
# Exact positive-P stochastic equations

$$
\begin{aligned}
d\alpha &= \{E(t) - [i\delta_k + i\chi(\beta + \beta^+) + \gamma_o]\alpha\}dt + dW_1,\\
d\beta &= [-(i\omega_m + \gamma_m)\beta - i\chi\alpha\alpha^+]dt + dW_2,\\
d\alpha^+ &= \{E^*(t) + [i\delta_k + i\chi(\beta + \beta^+) - \gamma_o]\alpha^+\}dt + dW_1^+,\\
d\beta^+ &= [(i\omega_m - \gamma_m)\beta^+ + i\chi\alpha\alpha^+]dt + dW_2^+,\\
d\alpha^{\text{out}} &= \sqrt{2\gamma_{\text{ext}}}d\alpha - d\alpha_{\text{ext}}^{\text{in}},\\
d\alpha^{\text{out}+} &= \sqrt{2\gamma_{\text{ext}}}d\alpha^+ - d\alpha_{\text{ext}}^{+\text{in}}. \qquad (2.8)
\end{aligned}
$$

**Internal photon and phonon modes, plus external input and output reservoirs are ALL included in the exact dynamical equations**

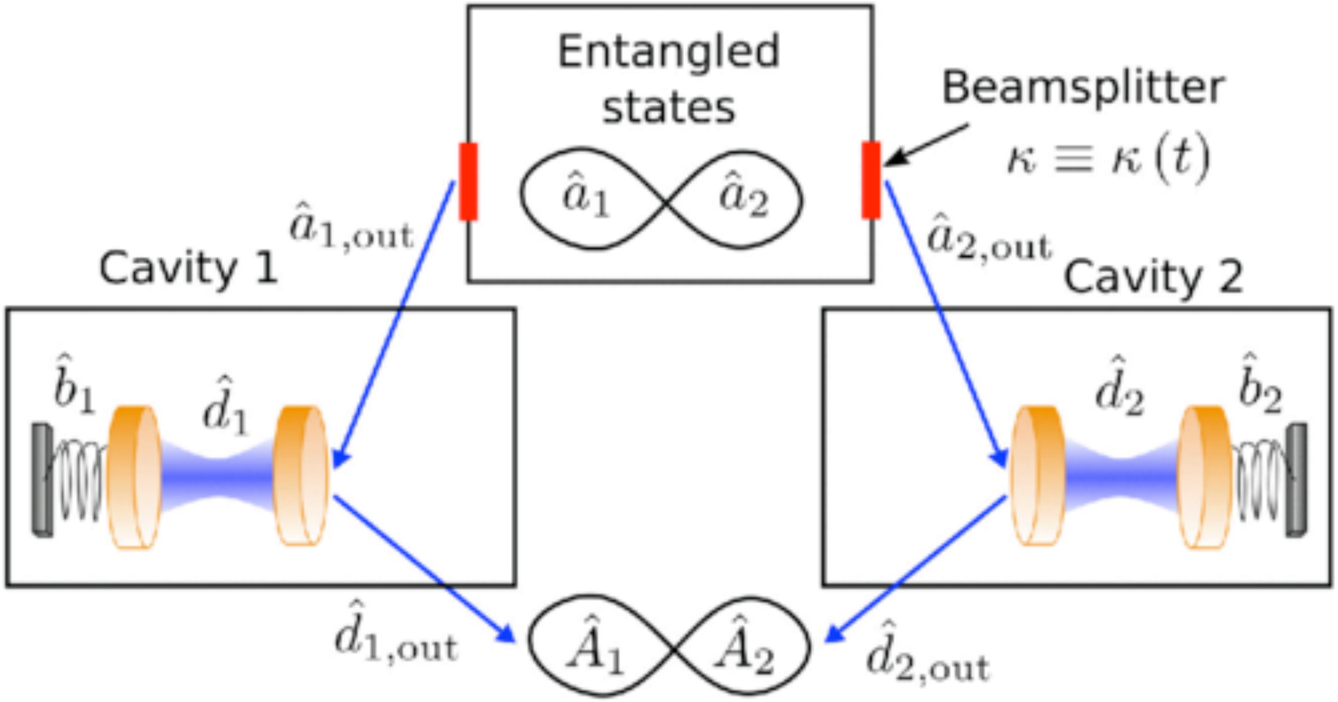
# Light and matter entanglement: theory vs JILA experiment

PHYSICAL REVIEW A **90**, 043805 (2014)




Data from: T.A. Palomaki, et. al., Science **342**, 710-713 (2013).

# Proposal: entangle two oscillators using a quantum memory




Q. Y. He, M. D. Reid, E. Giacobino, J. Cviklinski, P. D. D., PRA 79, 022310 (2009).
S. Kiesewetter, R. Y. Teh, P. D. D., and M. D. Reid, Phys. Rev. Lett. 119, 023601 (2017)

# Essential feature: temporal mode-matched input/output

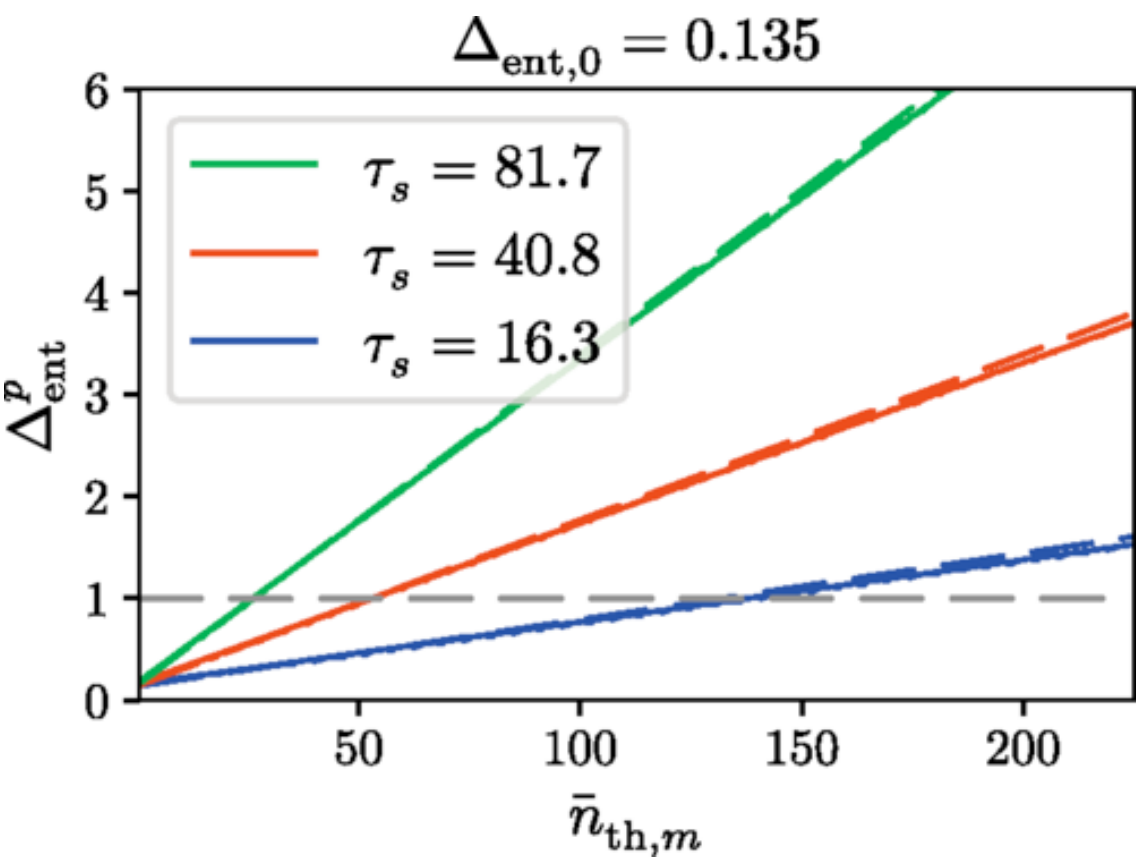
Must have temporal mode-matching
to ensure high-fidelity single-mode input

$$u_0^{in}(t) = -2i\frac{\sqrt{(\kappa_+ + m)(\kappa_+ - m)\kappa_+}}{m}\sinh(mt)\,e^{\kappa_+ t}\Theta(-t)$$

$$\text{where } \kappa_+ = (\gamma_o + \gamma_m)/2, \;\; \kappa_- = (\gamma_o - \gamma_m)/2$$

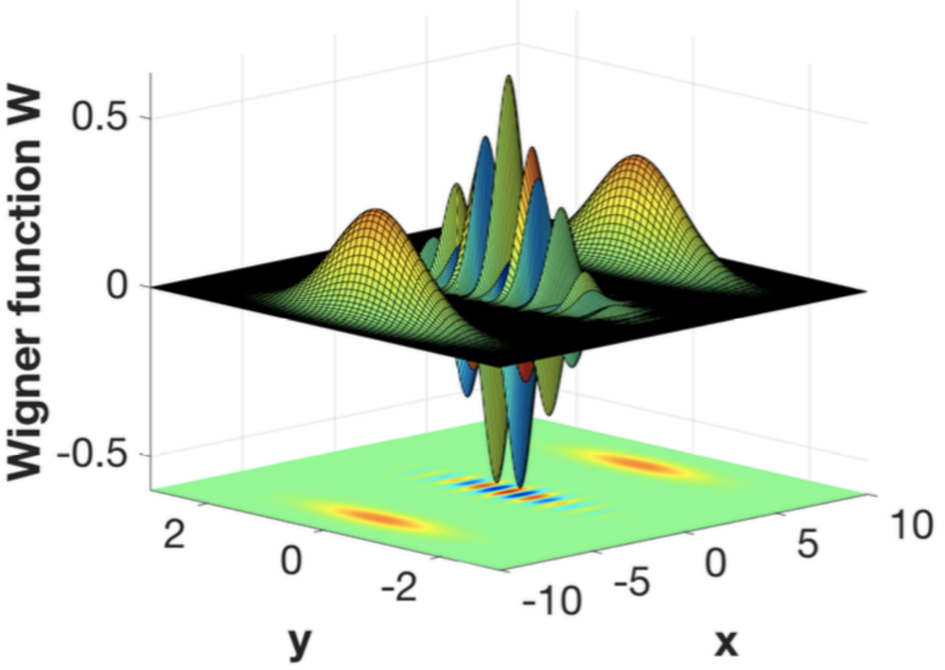
**This ensures perfect, temporally mode-matched input and output**

Predicted entanglement as a function of storage time



# Download photonic cat to a massive mechanical cat - see Yale experiments!

$$|\psi_{cat}\rangle = \frac{1}{\sqrt{\mathcal{N}}}(|\alpha_0\rangle + |-\alpha_0\rangle)$$




Note: this is a very pure cat!

# Schrodinger Cat predictions Phys. Rev. A 98, 063814 (2018).

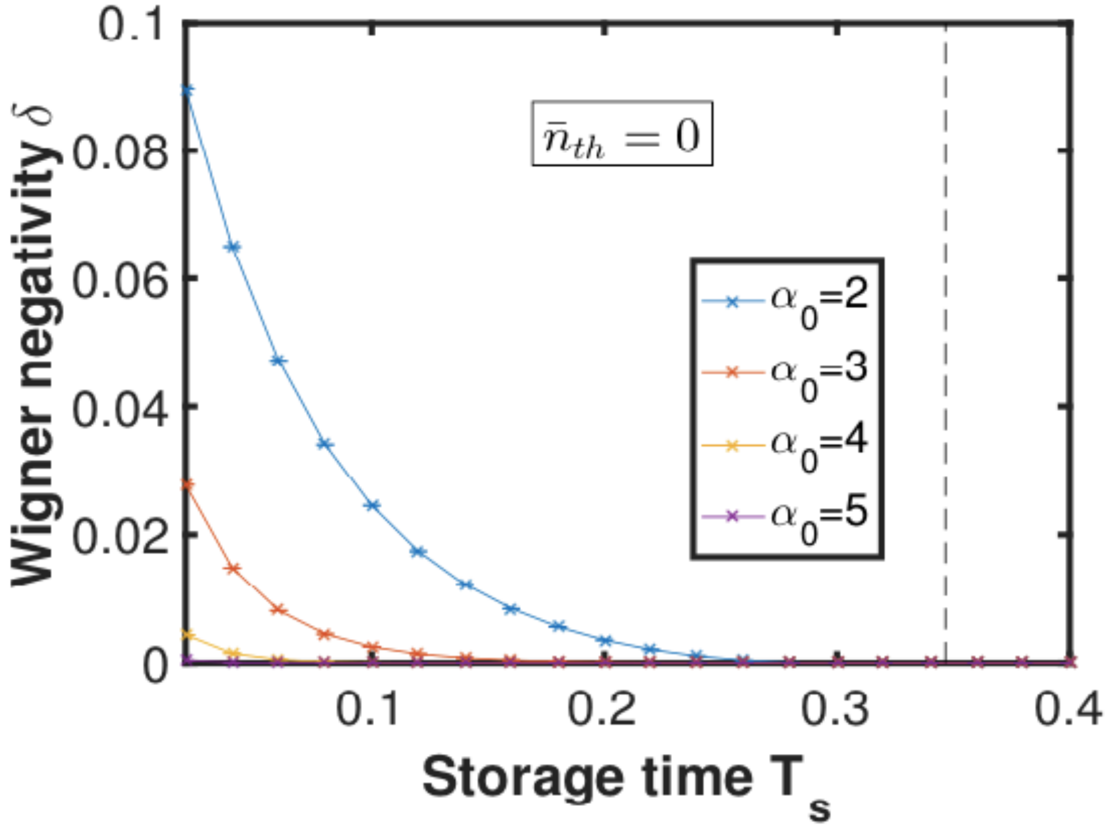
Input Schrodinger cat positive P-representation

$$P\left(\vec{\alpha}_0^{in}\right) = \frac{1}{\mathcal{N}}\left[\delta_{+,+} + \delta_{-,-} + e^{-2|\alpha_0|^2}\left(\delta_{+,-} + \delta_{-,+}\right)\right]$$

This is the input to the sampled equations, then used to calculate the output Wigner function of the stored cat state

$$W(\alpha) \approx \frac{2}{\pi N_s}\sum_{i}^{N_s} w\left(\vec{\alpha}_{0,i}^{in}\right) \mathrm{e}^{\left[-2\left(\alpha_{0,i}^{out+} - \alpha^*\right)\left(\alpha_{0,i}^{out} - \alpha\right)\right]}.$$

# Result of simulated mode-matched injection and retrieval




Parameters used are taken from:
('100 photon' CAT at Yale): C. Wang et al Science, 352, 1087 (2016).

# 4: BEC Schrodinger Cats

Rubidium experiment at SUT

Longest coherence time of any BEC interferometer

# Bose gas master equation, finite temperature

A $D$-dimensional Bose gas has two spin components that are linearly coupled by an external microwave field.

$$\hat{H} = \hbar \int d^3\mathbf{x} \left[ \frac{\hbar}{2m} \nabla \hat{\Psi}_i^\dagger \nabla \hat{\Psi}_i + V_i(\mathbf{x}) \hat{\Psi}_i^\dagger \hat{\Psi}_i + \frac{g_{ij}}{2} \hat{\Psi}_i^\dagger \hat{\Psi}_j^\dagger \hat{\Psi}_j \hat{\Psi}_i + \nu \hat{\Psi}_i^\dagger \hat{\Psi}_{3-i} \right]$$

Here, $g_{ij}$ is the self- and cross-coupling in $D-$dimensions. Collisional damping follows a master equation,

$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{\hbar} \left[ \hat{H}, \hat{\rho} \right] + \sum \kappa_\ell \int d^3\mathbf{x} \left[ 2\hat{O}_\ell \hat{\rho} \hat{O}_\ell^\dagger - \hat{O}_\ell^\dagger \hat{O}_\ell \hat{\rho} - \hat{\rho} \hat{O}_\ell^\dagger \hat{O}_\ell \right]$$

This includes self- and cross nonlinear damping, with

$$\hat{O}_\ell = \prod \hat{\Psi}_j^{\ell_j}$$

# Initial finite temperature state

- Take an initial finite temperature state
- Represent density matrix with Wigner
- Nonlinear chemical potential eliminates Bogoliubov `gapless' divergence problem
- King et. al., Journal of Physics A: 52, 035302 (2019).

$$\hat{K} = \hat{H} - \mu_1 \hat{N} - \frac{\mu_2}{2} \hat{N}^2$$

# Wigner phase-space: 1/N expansion

Result of Wigner operator mappings:

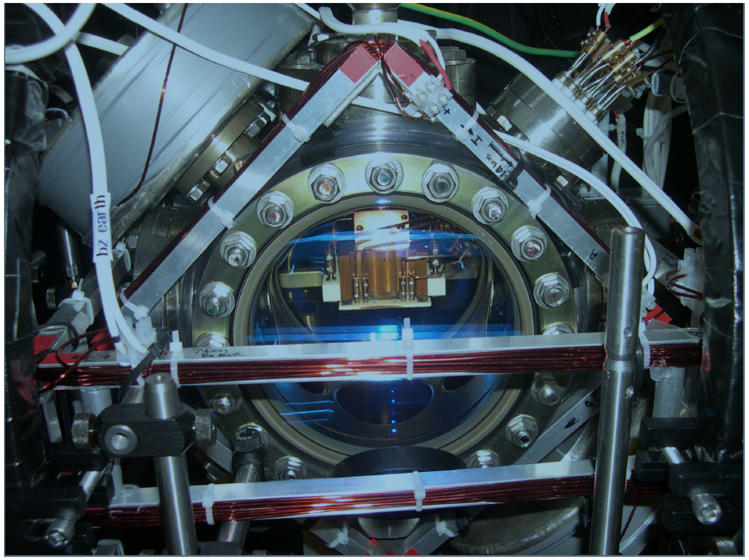
$$i\partial_\tau \psi_i = \left\{ -\frac{1}{2}\nabla_\zeta{}^2 + \gamma\psi_i^\dagger\psi_i + \gamma_c\psi_j^\dagger\psi_j \right\}\psi_i - \tilde{\nu}\psi_j,$$

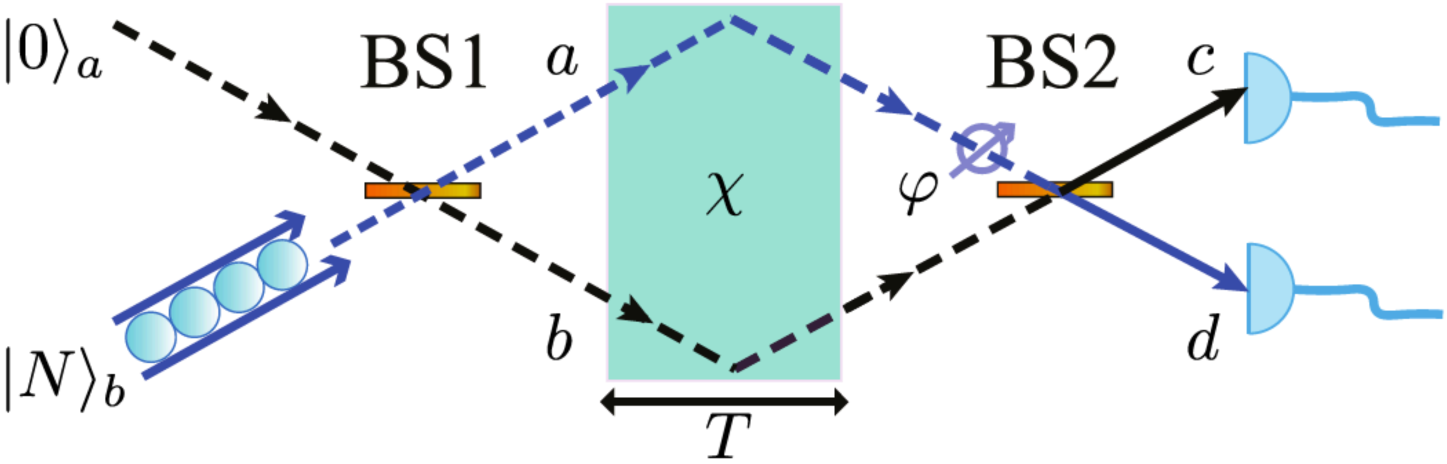
$$-\sum \tilde{\kappa}_\ell \frac{\partial \tilde{O}_\ell^*}{\partial \psi_i^*}\tilde{O}_\ell + B_{ij}[\boldsymbol{\psi}]\,\eta_j(t,x)$$

Scaling: $\tau = t/t_0$, $\zeta = x/x_0$,

$t_0 = \hbar/gn$; $x_0 = \hbar/\sqrt{gnm}$; $\langle \Delta\tilde{\psi}(\zeta)\Delta\tilde{\psi}^*(\zeta')\rangle = \frac{1}{2}\delta(\zeta - \zeta')$.

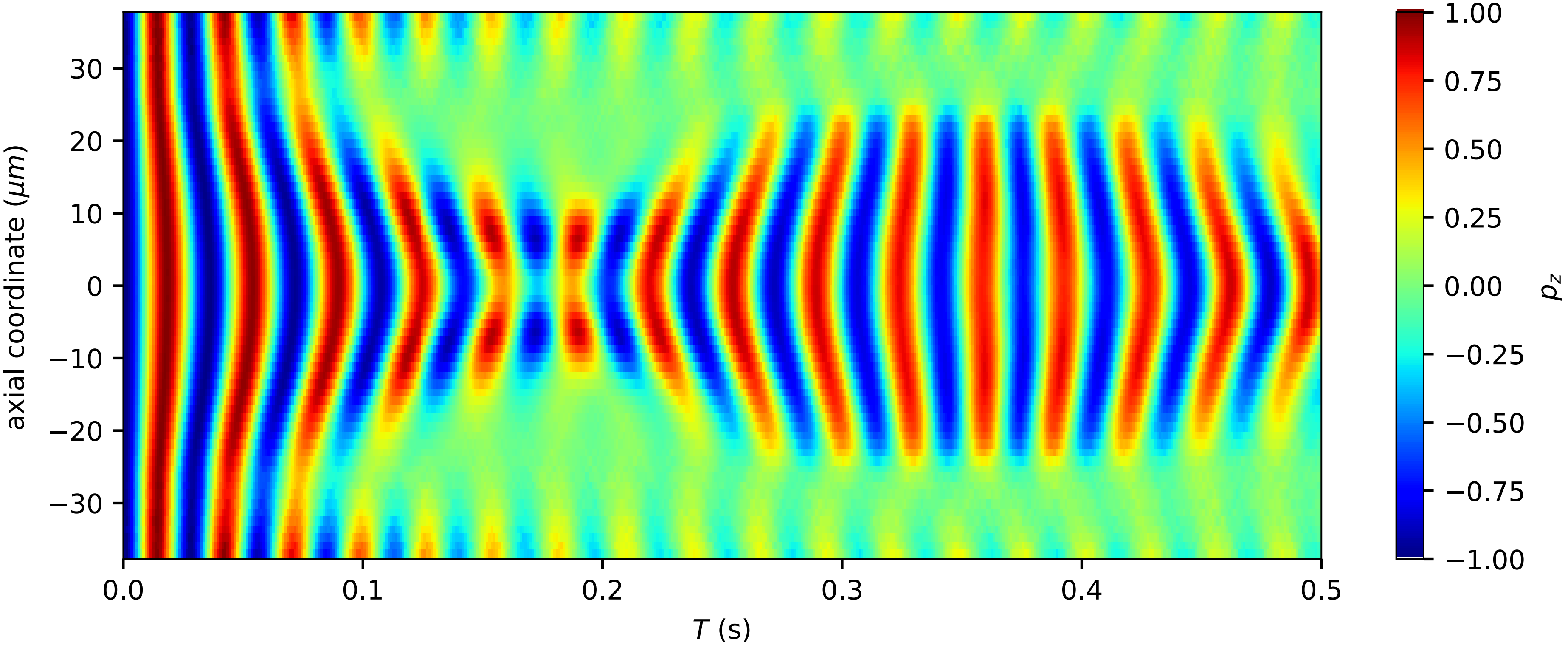
# Test case: Interferometry on an atom chip (Sidorov, Swinburne)



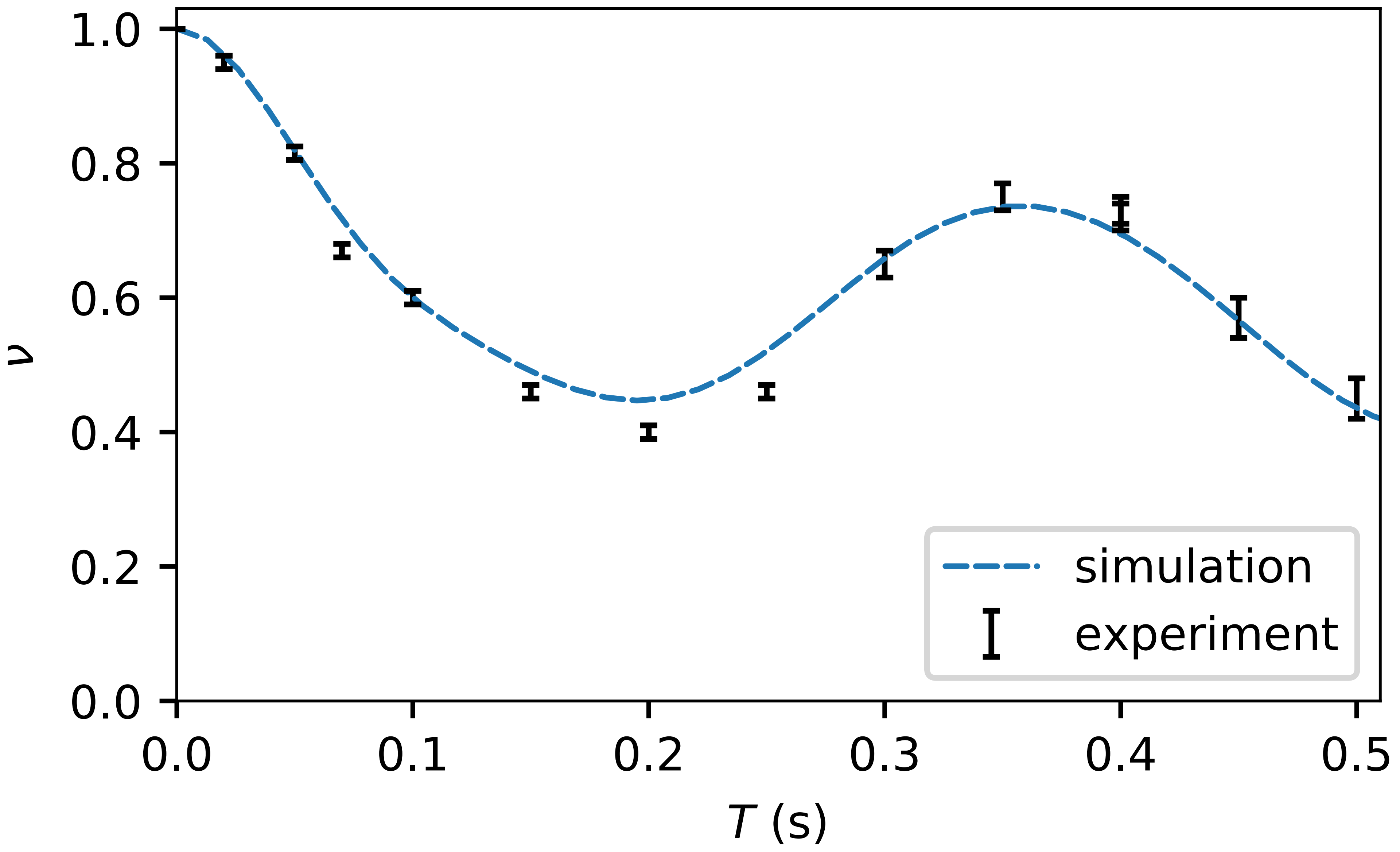



# Rubidium interferometry

A two-component, $4 \times 10^4$ atom $^{87}Rb$ BEC is in a harmonic trap with internal Zeeman states $|1, -1\rangle$ and $|2, 1\rangle$, which can be coupled via an RF field.

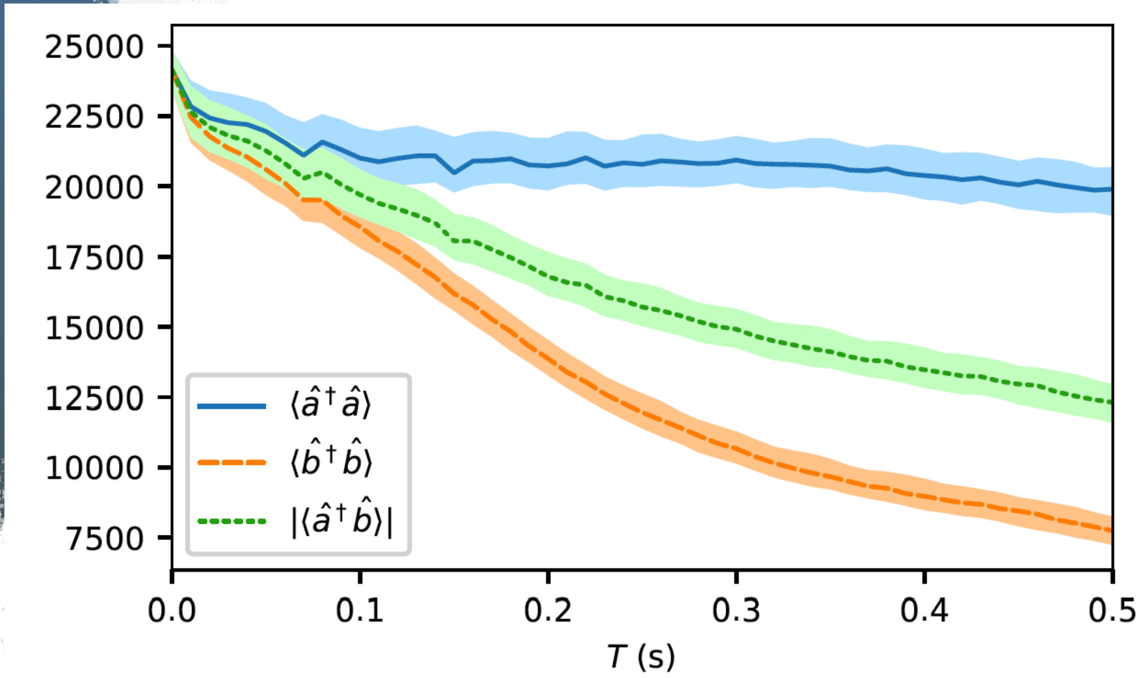


# Computed vs observed 3D fringe visibility



# Evidence for 40,000 atoms entangled

- Calculate dynamical condensate occupation
- Combine with fringe visibility
- Evidence for macroscopic entanglement



# 5: 1D Bose gas breathers

Joint program with UMass, Tel Aviv,

Experiments at Rice U.

**King Ng, Bogdan Opanchuk, Margaret D. Reid, P.D.D.,**

**Phys. Rev. Lett. 122, 20364, 2019**

# One-dimensional Bose gas

## Hamiltonian

$$\hat{H}_{1\mathrm{D}} = \int \hat{\Psi}^{\dagger}_{1\mathrm{D}} H_1 \hat{\Psi}_{1\mathrm{D}} dr_3 + \frac{g_{1\mathrm{D}}}{2} \int \left(\hat{\Psi}^{\dagger}_{1\mathrm{D}}\right)^2 \hat{\Psi}^2_{1\mathrm{D}} dr_3$$

$$H_1 = -\hbar^2 \partial_3^2/2m + m\omega_3^2 r_3^2/2$$

$$g_{1\mathrm{D}} = 2\hbar\omega_\perp a$$

↓

$$r_0^2 = \hbar t_0/2m$$

↓

$$z = r_3/r_0 ; \hat{\psi} = \sqrt{r_0}\hat{\Psi}_{1\mathrm{D}} ; \tau = t/t_0 ; \hat{\psi}_{,z}(z) \equiv \partial_z \hat{\psi}(z)$$

↓

$$\hat{H} = \int dz \left[\hat{\psi}^{\dagger}_{,z}(z)\hat{\psi}_{,z}(z) + C\left(\hat{\psi}^{\dagger}(z)\right)^2 \hat{\psi}^2(z)\right]$$

$$C = m g_{1\mathrm{D}} r_0/\hbar^2$$

# Conservation laws



Local symmetry from Noether's theorem leads to globally conserved quantities

1. Particle number $\hat{N} = \sum_{k,} \hat{n}_k$
2. Momentum $\hat{P} = \sum_{k,} k\hat{n}_k$
3. Energy $\hat{H} = \sum_{k,} k^2\hat{n}_k + \frac{C}{V}\sum_{\boldsymbol{k}} \hat{a}^\dagger_{k_1}\hat{a}^\dagger_{k_2}\hat{a}_{k_3}\hat{a}_{k_4}\delta_{\boldsymbol{k}}$
4. Higher order term $\hat{H}_3 = \sum_{k,} k^3\hat{n}_k + \frac{3C}{2V}\sum_{\boldsymbol{k}} (k_1 + k_2)\,\hat{a}^\dagger_{k_1}\hat{a}^\dagger_{k_2}\hat{a}_{k_3}\hat{a}_{k_4}\delta_{\boldsymbol{k}}.$

**Quench experiment:**

- Make an attractive soliton, increase coupling by 4x
- Exact solutions, DMRG – fail at N>5

# Conservation laws: Truncated Wigner



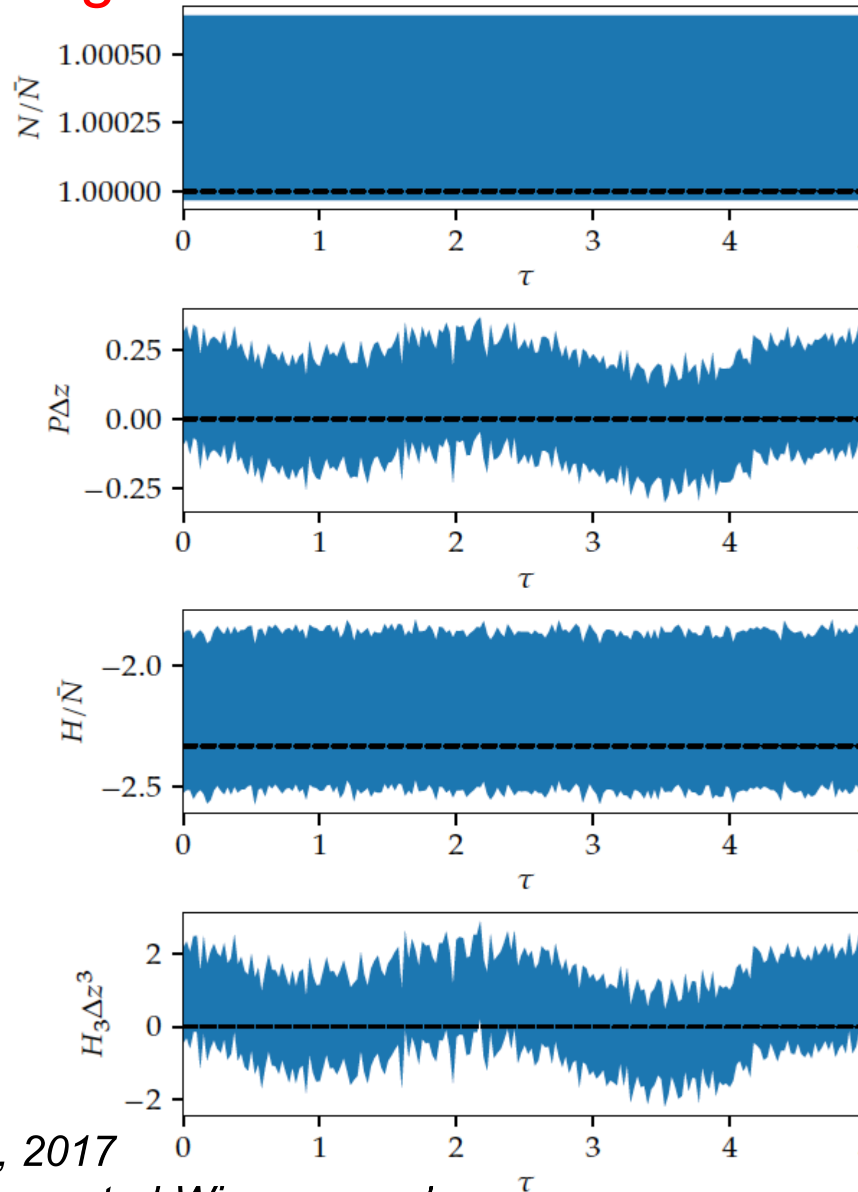
- Conservation of quantities in Wigner representation

$$\left\langle \hat{N} \right\rangle_W = \langle N \rangle_W - \frac{1}{2} M_0$$

$$\left\langle \hat{P} \right\rangle_W = \langle P \rangle_W$$

$$\left\langle \hat{H} \right\rangle_W = \left\langle H - \frac{2C}{\Delta z} N \right\rangle_W - \frac{1}{2} M_2 + \frac{MC}{2\Delta z}$$

$$\left\langle \hat{H}_3 \right\rangle_W = \left\langle H_3 - \frac{3C}{\Delta z} P \right\rangle_W$$

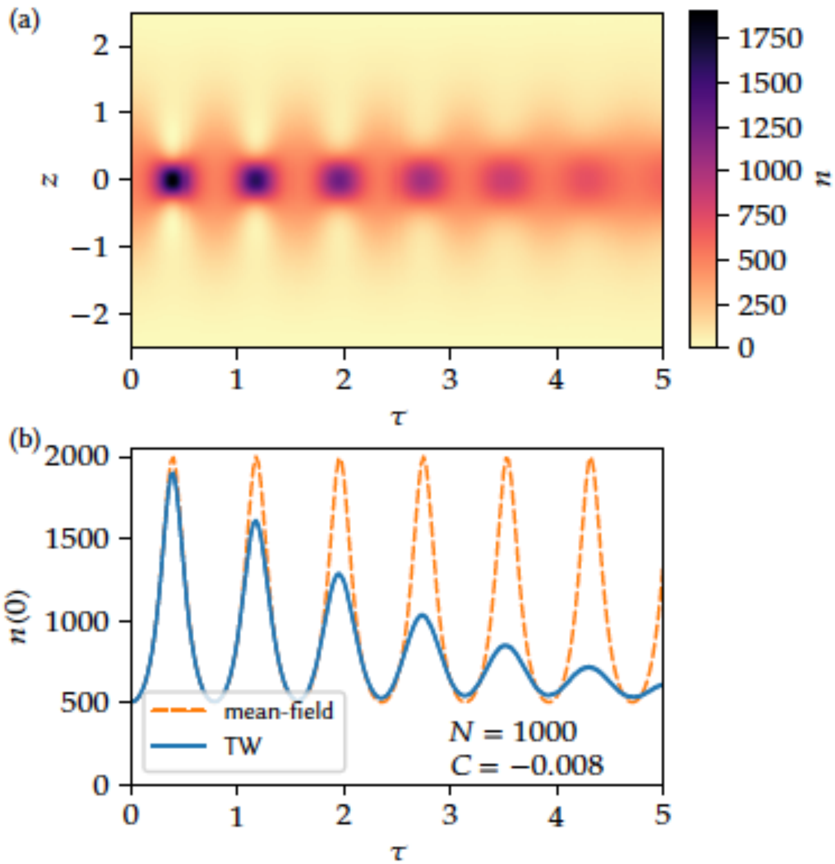


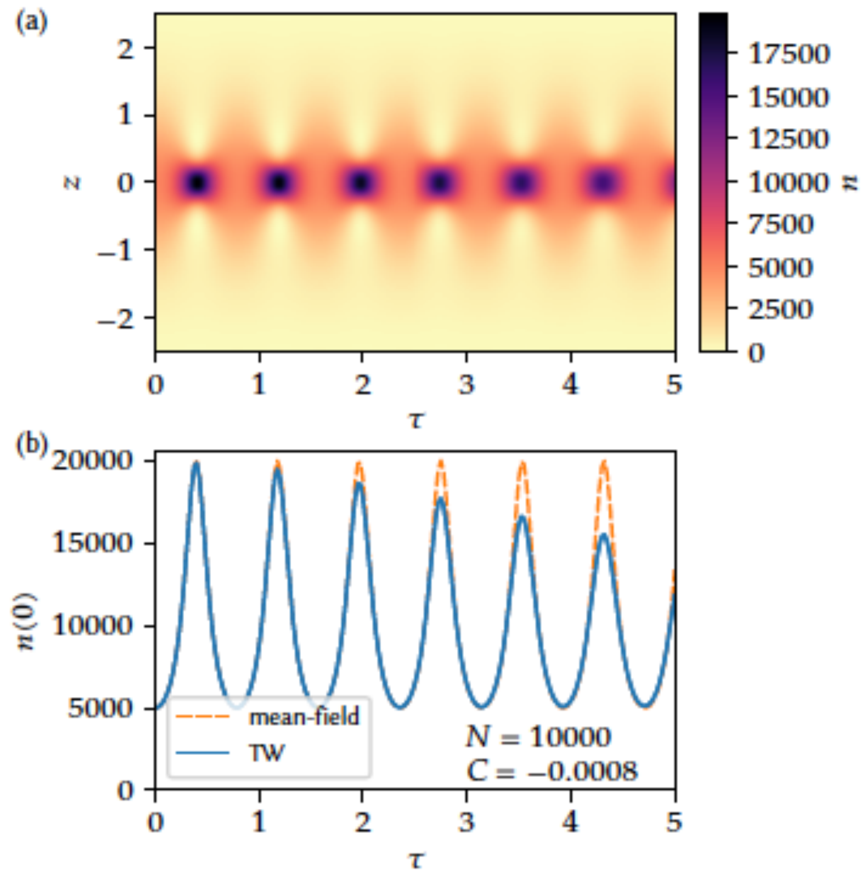

*Ref: Physical Review A 96, 043616, 2017*

*Black:exact classical result; blue: Truncated-Wigner error bars*

# Truncated Wigner: breather relaxation





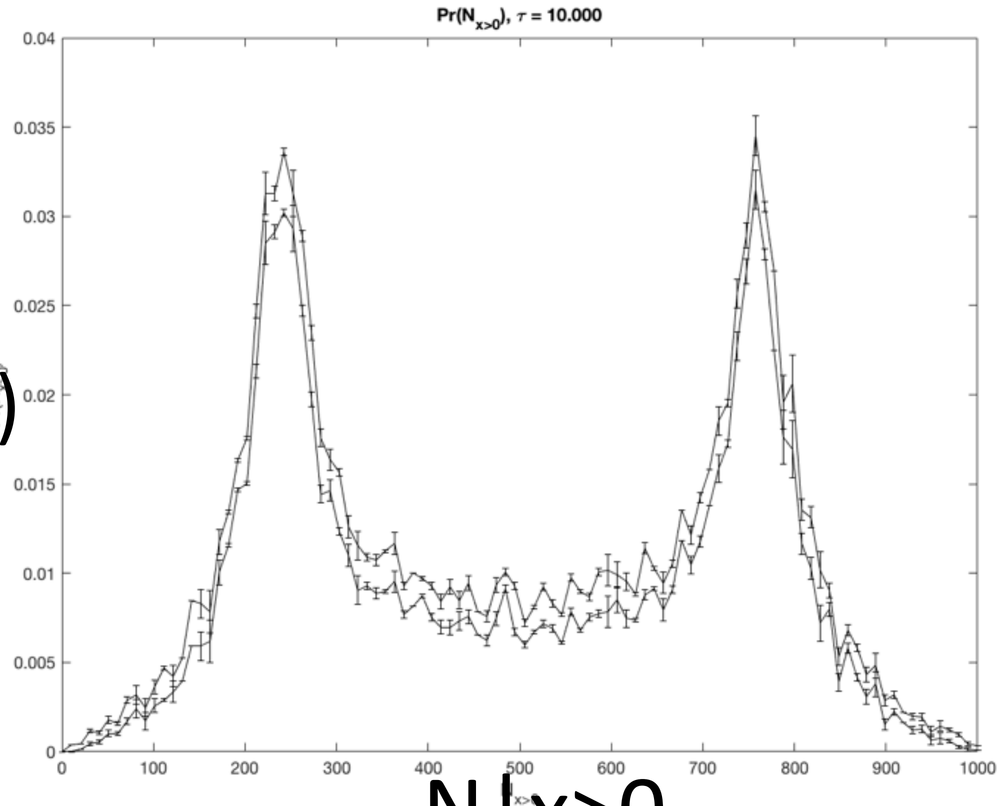




*Ref: Phys. Rev. A 96, 053628, 2017*

- Gradual fragmentation
- Decay of breather

# Multimode Schrodinger cat

## Soliton splits either way, 3:1 number ratio




P(N|x>0)

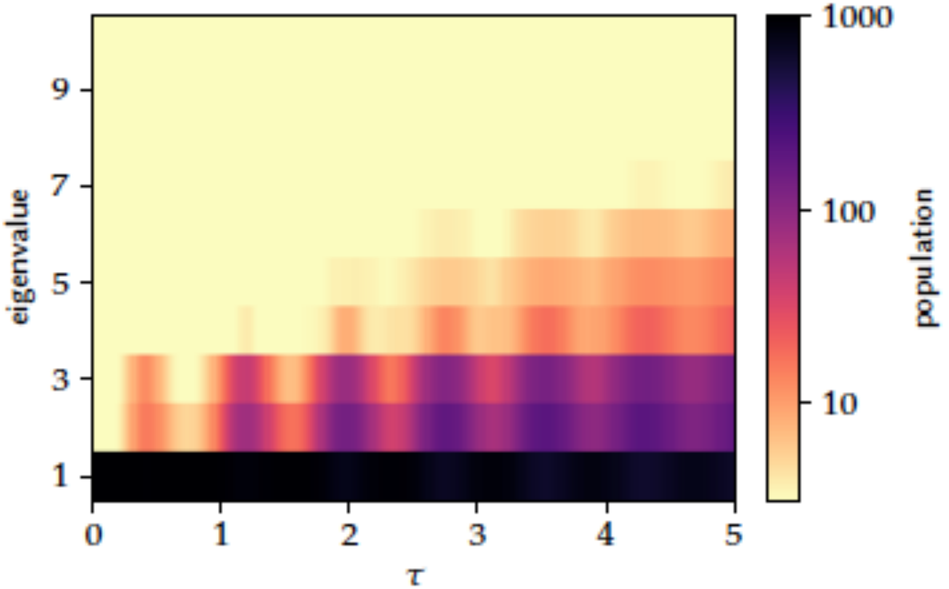
N|x>0

Also see: **Yurovsky et. al, Phys. Rev. Lett. 119, 220401 (2017)**

# Truncated Wigner: multi-mode evolvement

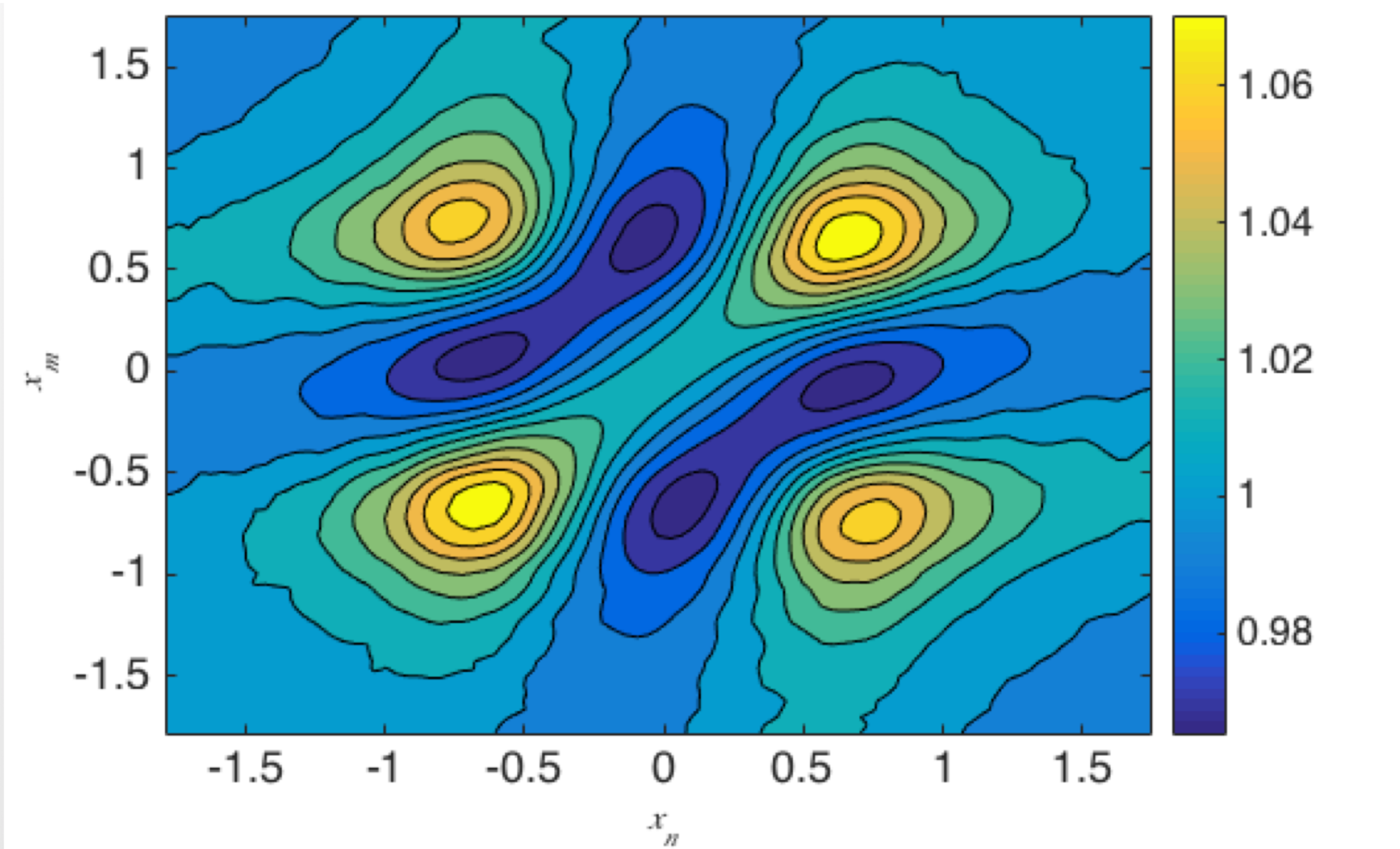


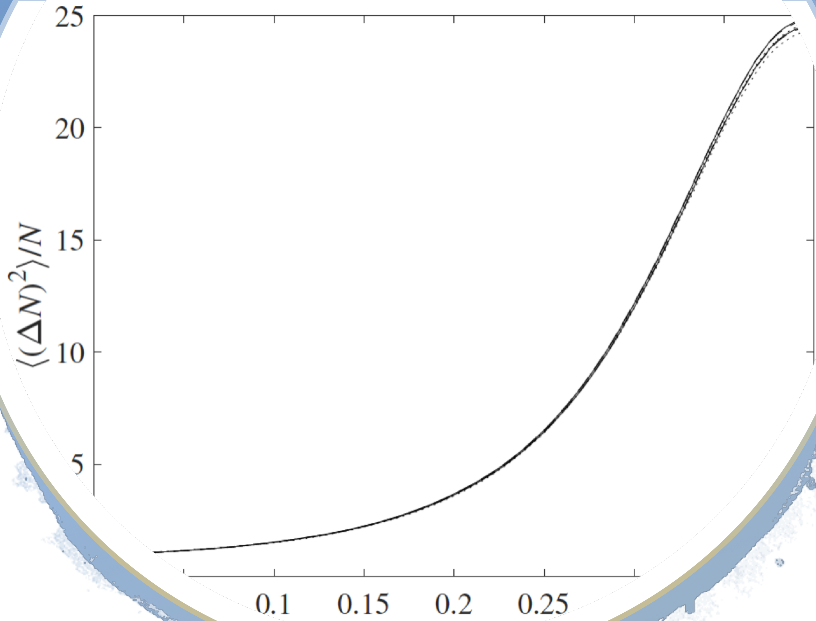
- Single eigen-mode evolves to multi-eigenmode (~7)
- Partial fragmentation

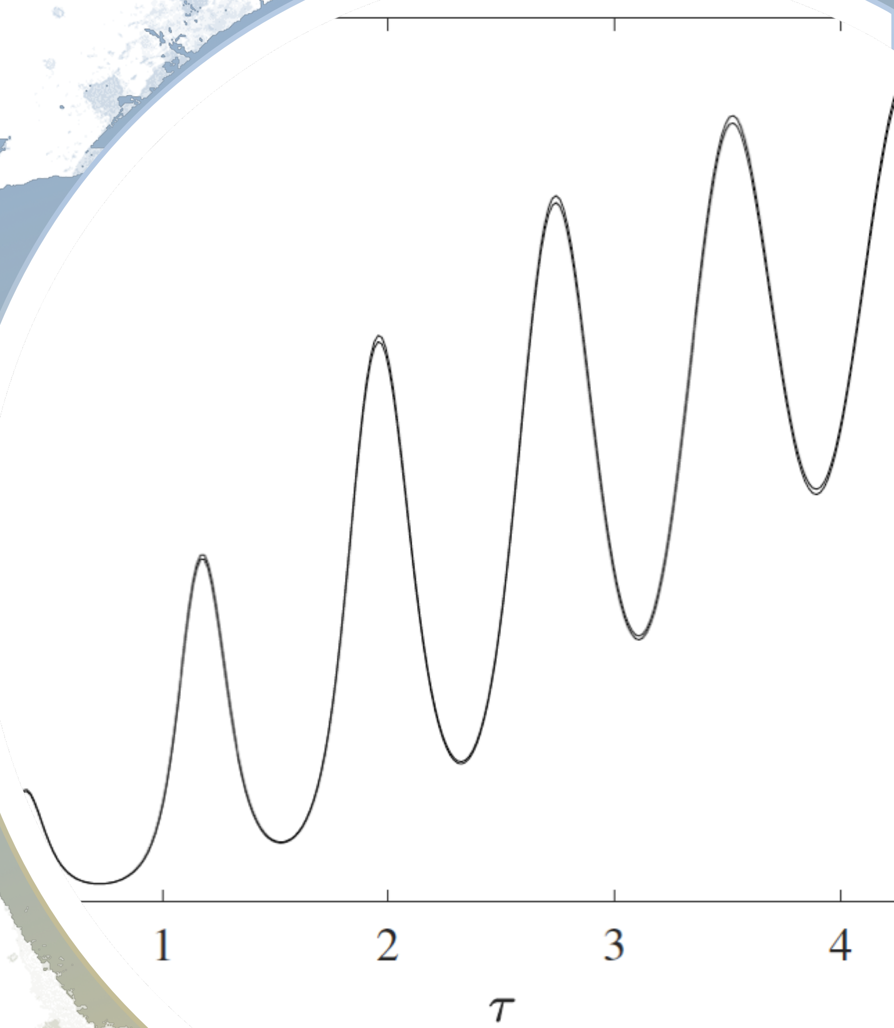



*Ref: Phys. Rev. A 96, 053628, 2017*

# Second-order correlation: $g^{(2)}(x_m,x_n)$






**Time-evolution of left-right number difference.**

Agreement of exact +P and truncated Wigner state, with *either* number state *or* Poissonian initial conditions.

**Experiments at Rice U.**

# 6: Early universe simulations

Quantum field theory: exponentially complex

Essential to current theories of cosmology

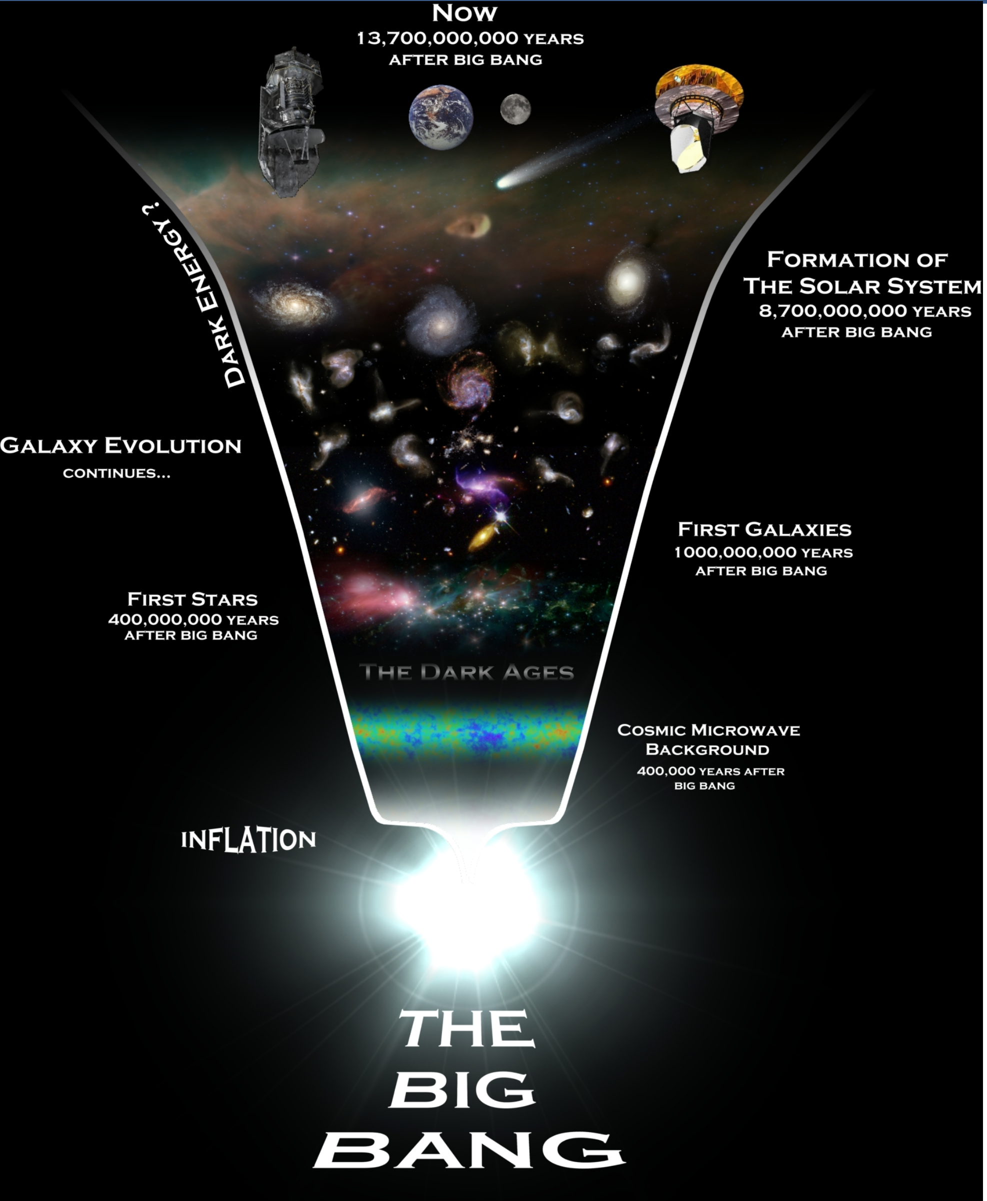
Energies a trillion times larger than CERN

How can we compute what theory predicts?

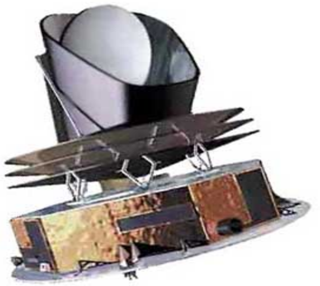
**Use ultracold BEC as relativistic simulator**

Check predictions with computer simulations

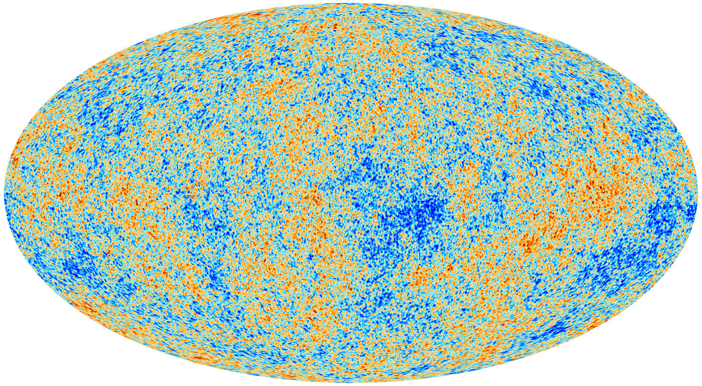
# How can we test theories of the Big Bang?




Planck spacecraft was launched in May 2009. On 21 March 2013, the mission's all-sky map of the CMB was released



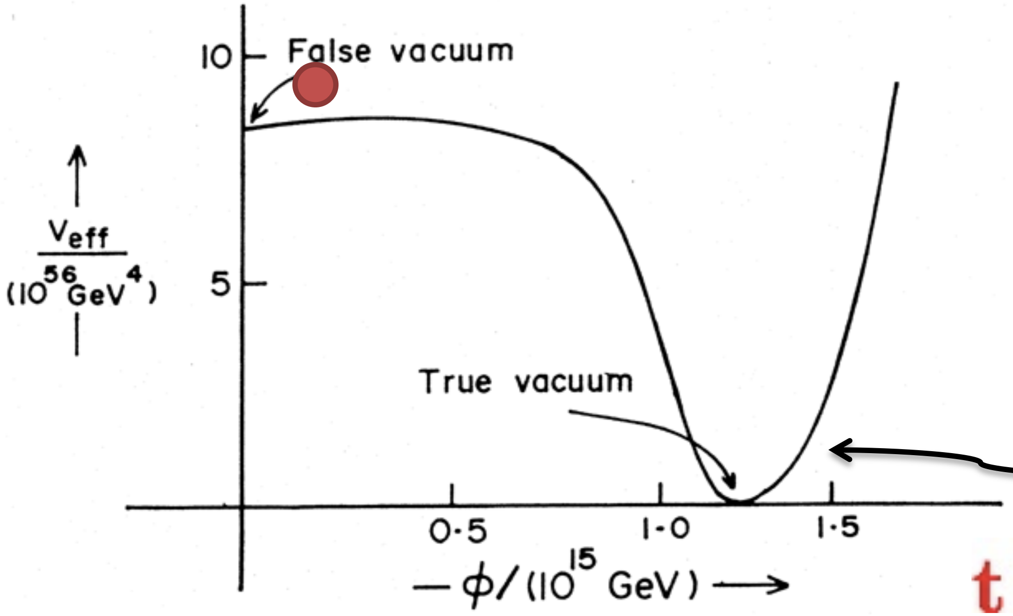
The CMB is a snapshot of the oldest light in our Universe, imprinted on the sky when the Universe was just 380,000 years old.



We can't see beyond that

BGV theorem

# Quantum models of the Big Bang




Potential energy on top of the hill is converted into kinetic energy of the rolling ball at the bottom of the hill.

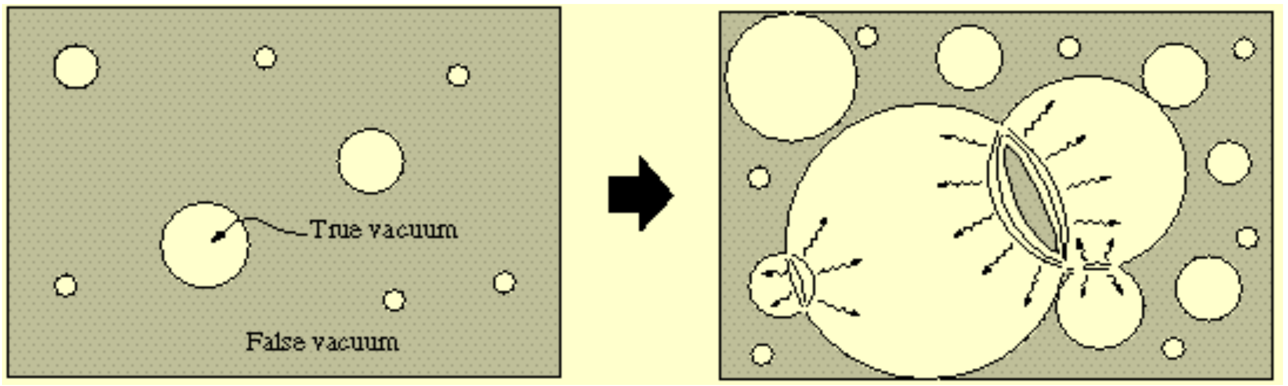
As a result a lot of energy was realised resulted in

**BIG BANG**

$t \sim 10^{-32} sec$

In reality the Universe has at least 3 dimensions. Bubbles appear during the transition to true vacuum.

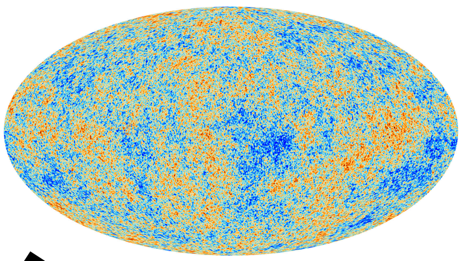
Are we in one of the bubbles….lonely….?

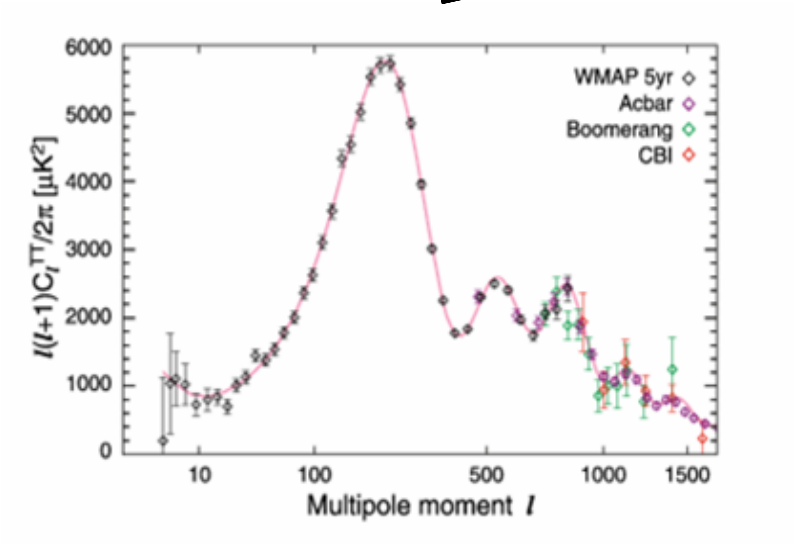



Similar to water boiling or bubbles in champagne

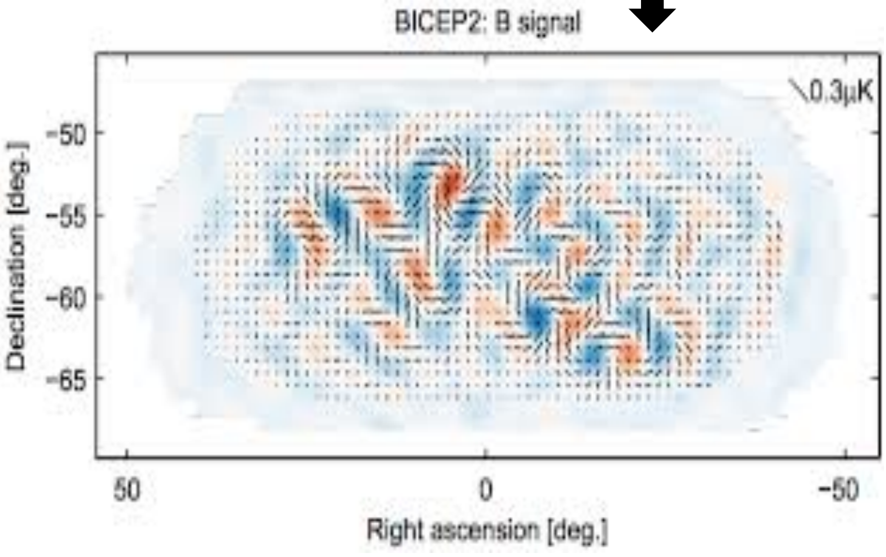
# What is the observational evidence?

Planck spacecraft,
21 March 2013



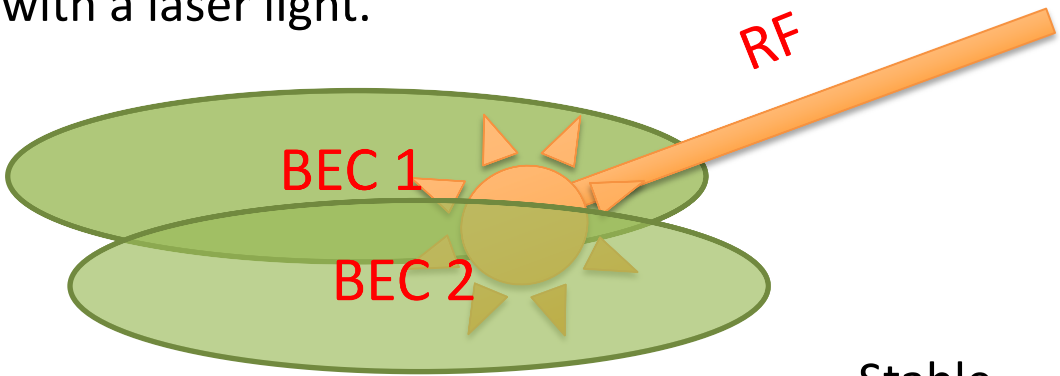



Bicep 2, Polar Bear,
South Pole Telescope



# Analog quantum simulator

Take 2 BECs and couple them with a laser light.

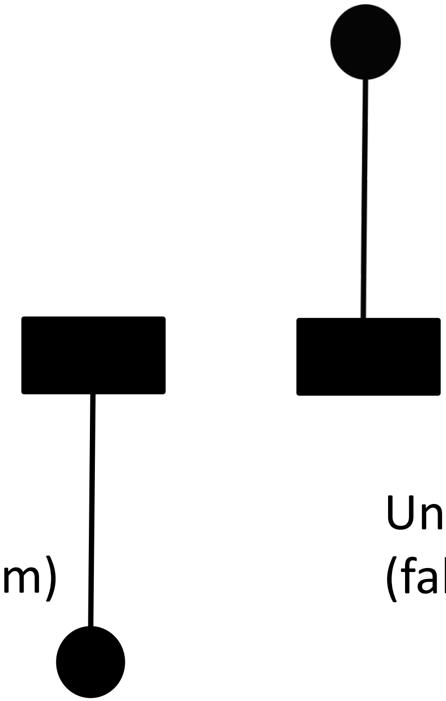



$$\phi_1 - \phi_2 \approx \theta$$

Their phase difference behaves like a pendulum, which has stable and unstable points.

$$\partial_t^2 \theta - c^2 \nabla^2 \theta = -\partial_\theta V(\theta)$$

-- relativistic field equation of the early Universe.

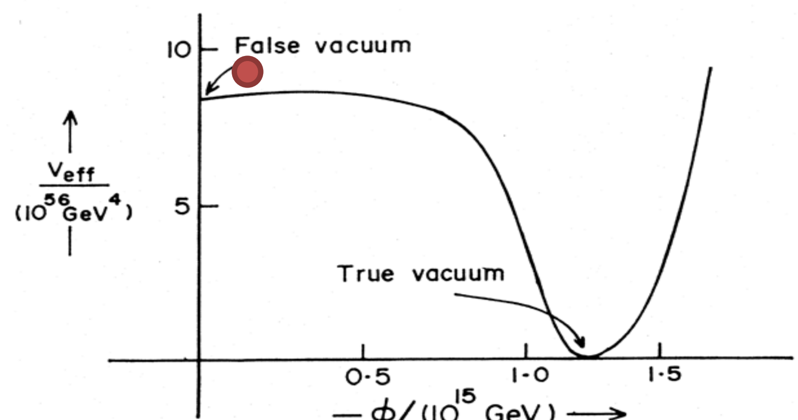


Stable (true vacuum)

$\theta = 0$

Unstable (false vacuum)

$\theta = \pi$



# Early universe models

- The simplest model has a scalar inflaton field
- Relativistic, interacting quantum field dynamics
- $\phi(x)$ is described by the Lagrangian

$$\mathscr{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi),$$

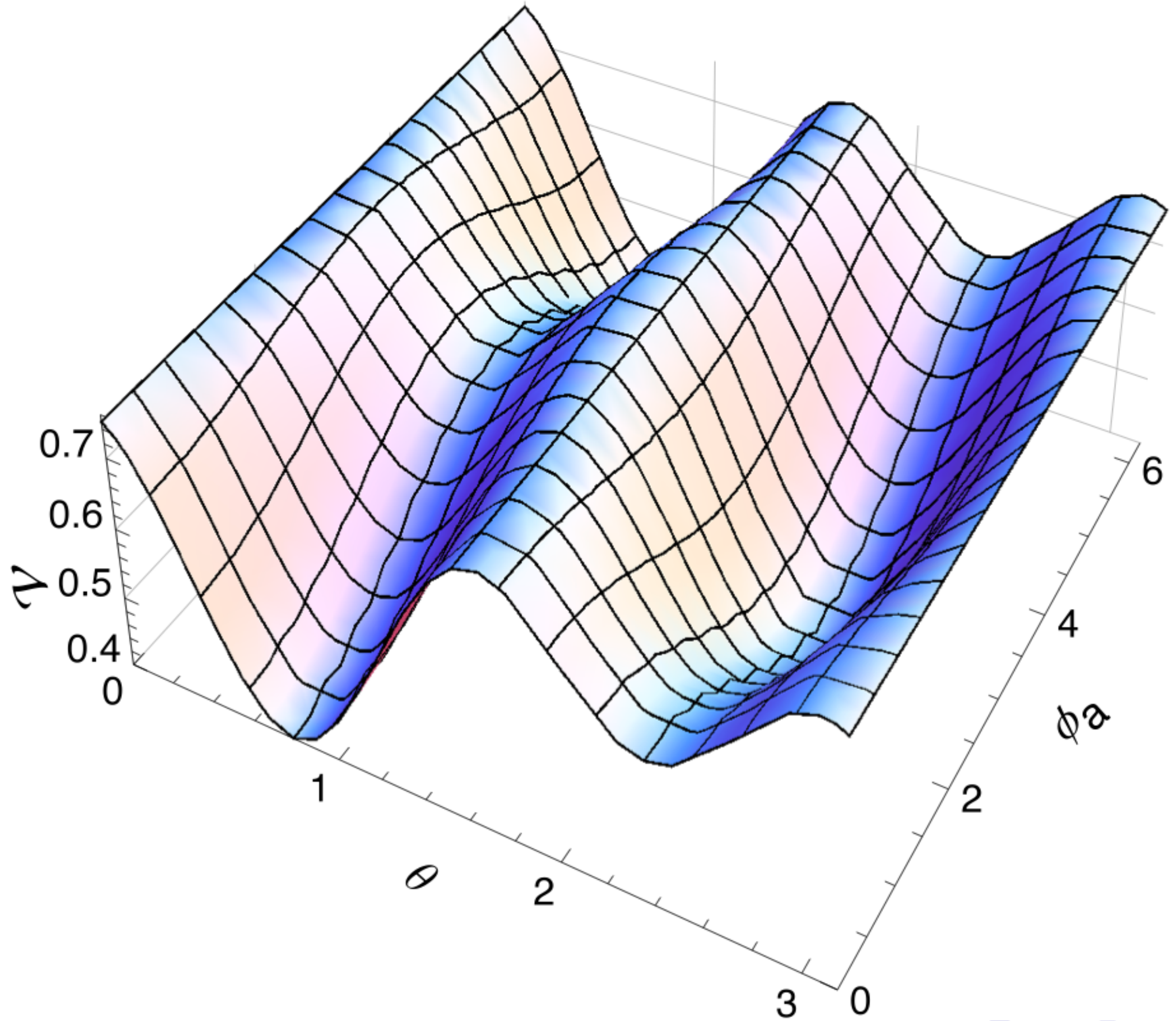
where $V(\phi)$ is the potential down which the scalar field rolls

# Early universe quantum simulation

## $^{41}K$ Feshbach resonance

- zero inter- state scattering length at 685.7 G
  - nearly equal self-interactions,
  - unknown loss rates (can be estimated)
  - resonance not yet observed

# Potential well with microwave coupling



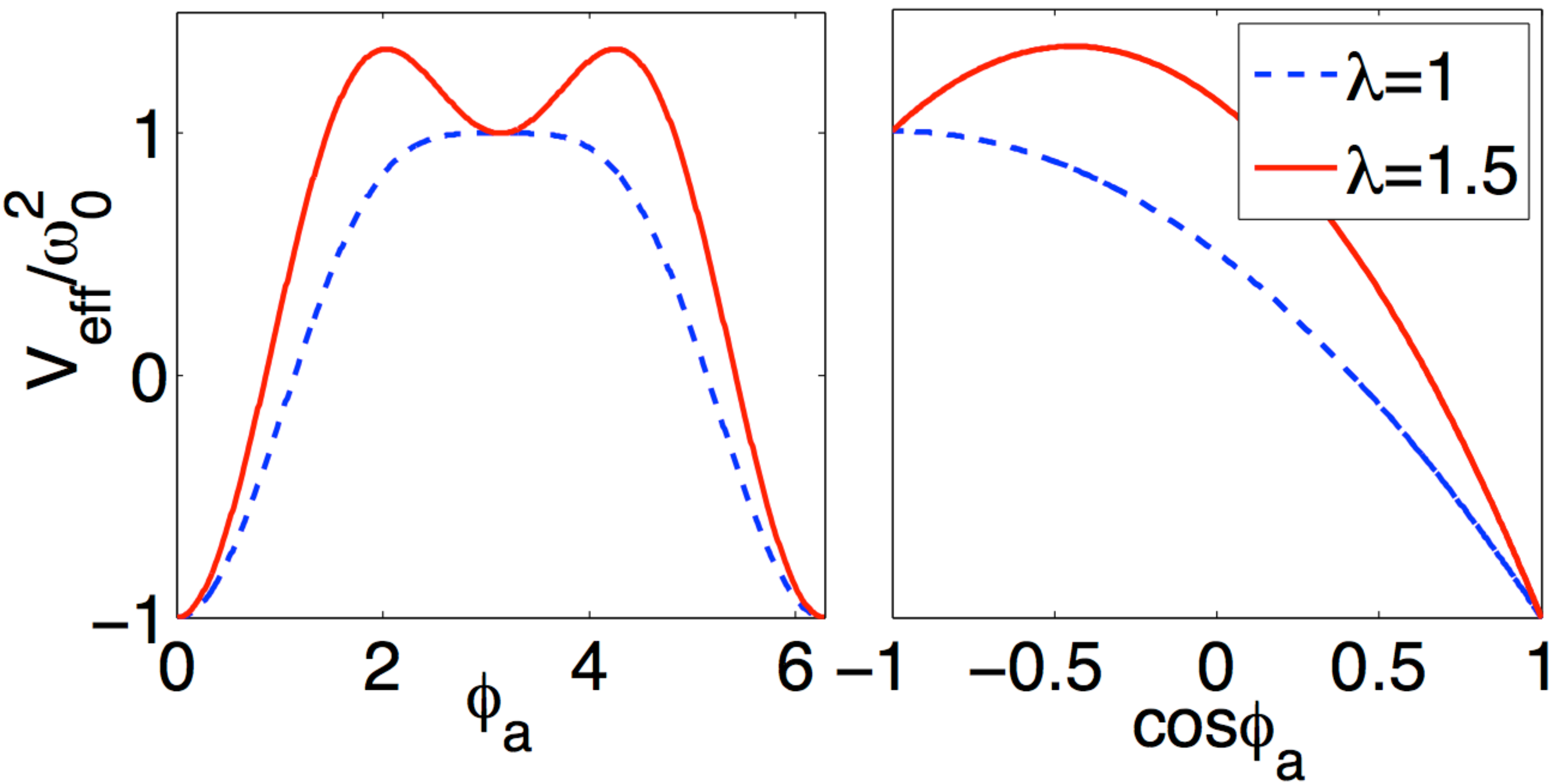
# Equivalent Sine-Gordon equation

$$\psi_1 = u e^{i(\phi_s+\phi_a)/2}\cos(\theta)$$
$$\psi_2 = u e^{i(\phi_s-\phi_a)/2}\sin(\theta),$$

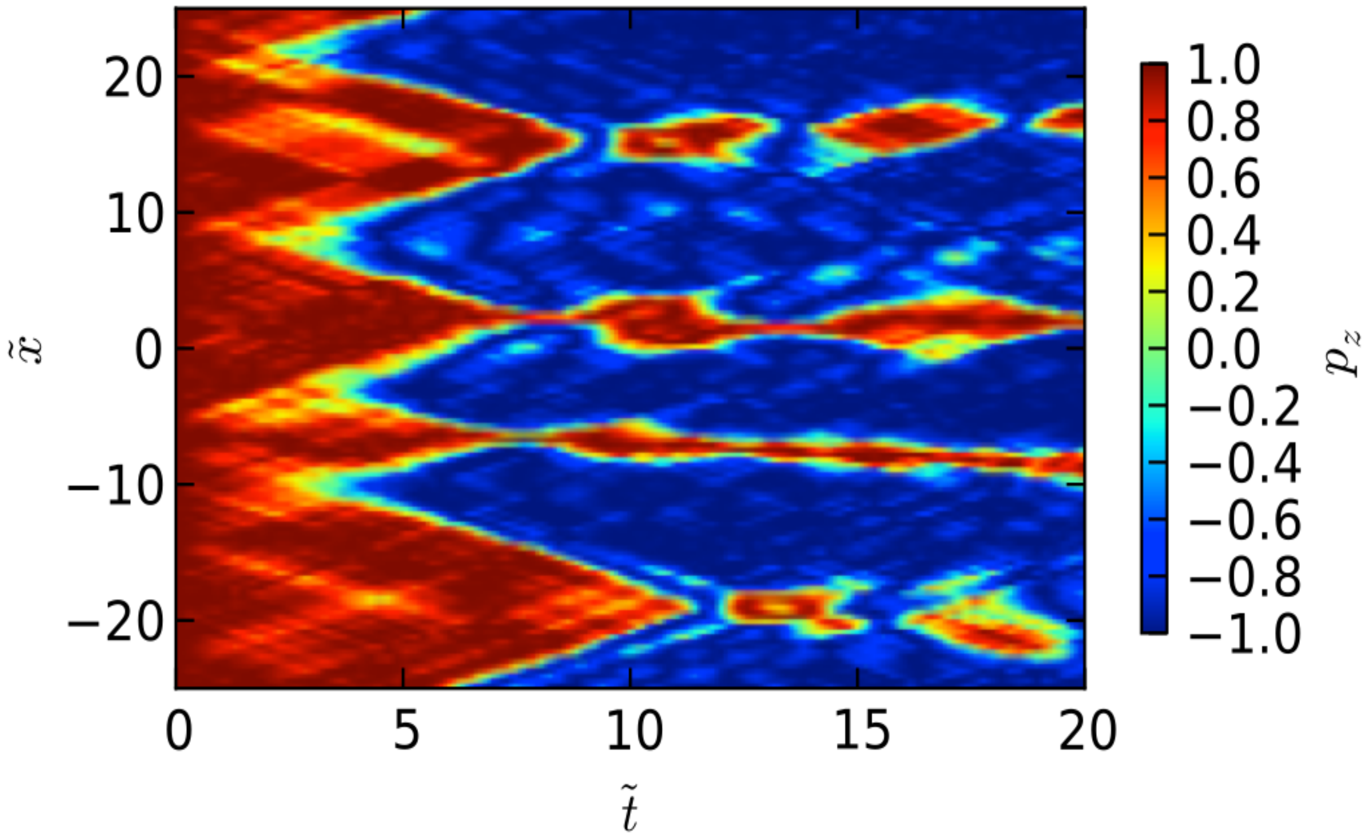
- Canonical momentum: $\pi = \partial_\tau \phi_a / 4\gamma_{sa}$ ,
- Commutators: $[\phi_a(\zeta), \pi(\zeta')] = i\delta^D(\zeta - \zeta')$ .
- Sine-Gordon equation:

$$\nabla^2\phi_a - \partial_{\zeta_0\zeta_0}\phi_a + \tilde{\alpha}\sin\phi_a = 0$$

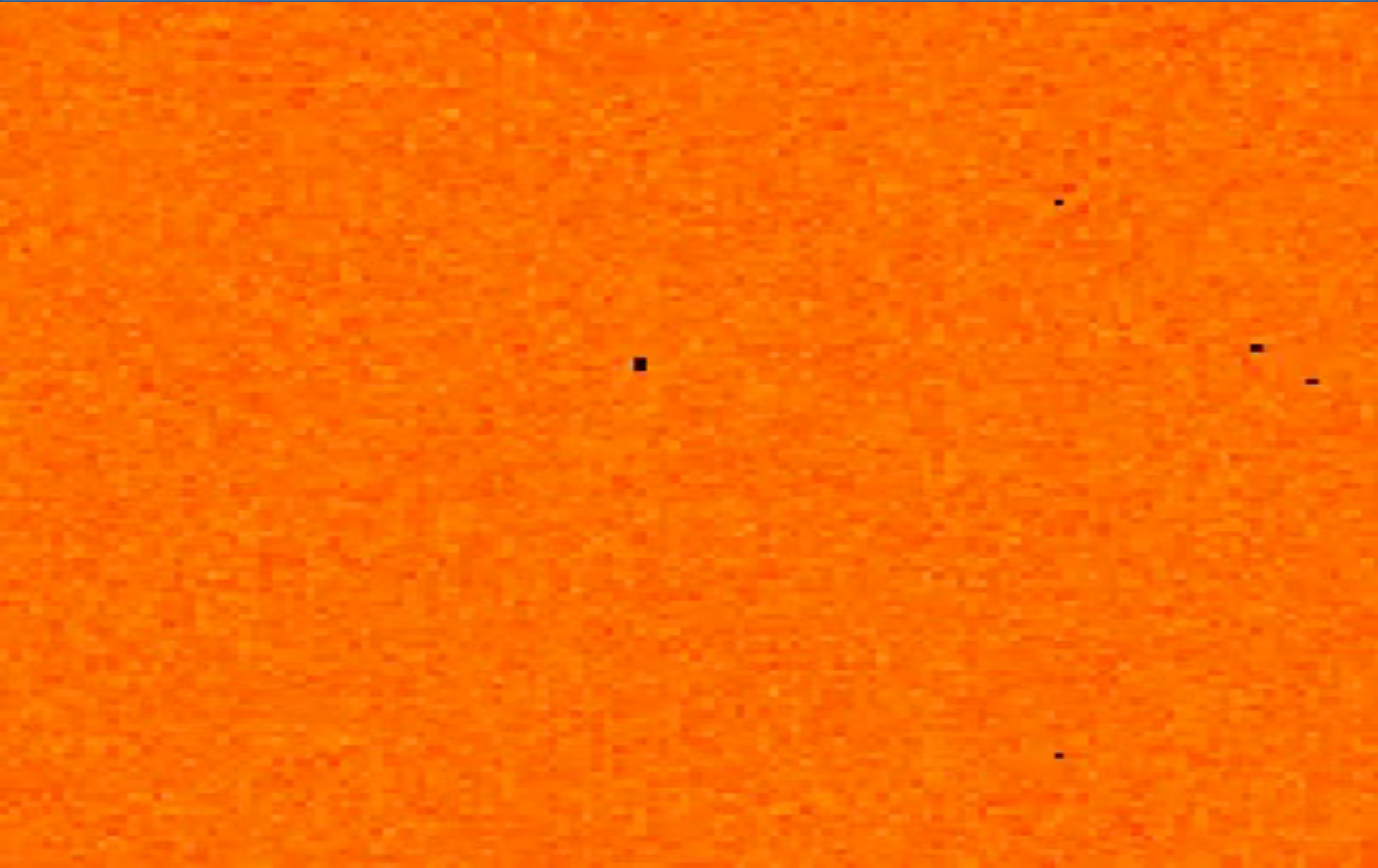
# Effective potential



# Vacuum bubbles expand at light-speed



# Metastable 2D Universe: BEC simulations



# SUMMARY

## Positive P-representation

Exact intracavity open quantum dynamics, opto-mechanics, Schrodinger cats

## Complex P-representation

Exact Boson sampling quantum simulations –large mode numbers, huge permanents

## Wigner representation

Treatment of large BEC systems with 1/N expansion, millions of modes possible

## Next step:

Stochastic bridges, interacting Fermi phase space